A Model for Rolling Bearing Life with Surface and Subsurface Survival — Tribological Effects

Guillermo E. Morales-Espejel, Antonio Gabelli and Alexander J. C. de Vries

Until now the estimation of rolling bearing life has been based on engineering models that consider an equivalent stress, originated beneath the contact surface, that is applied to the stressed volume of the rolling contact. Through the years, fatigue surface–originated failures, resulting from reduced lubrication or contamination, have been incorporated into the estimation of the bearing life by applying a penalty to the overall equivalent stress of the rolling contact. Due to this simplification, the accounting of some specific failure modes originated directly at the surface of the rolling contact can be challenging. In the present article, this issue is addressed by developing a general approach for rolling contact life in which the surface-originated damage is explicitly formulated into the basic fatigue equations of the rolling contact. This is achieved by introducing a function to describe surface-originated failures and coupling it with the traditional, subsurface-originated fatigue risk of the rolling contact. The article presents the fundamental theory of the new model and its general behavior. The ability of the present general method to provide an account for subsurface-originated fatigue risk of the rolling contact. The article presents the fundamental theory of the

**Nomenclature**

- $A$ = Damage risk area (m$^2$)
- $\bar{A}$ = Constant
- $\bar{A}_c$ = Damage constant for volume, damage constant for a rolling contact
- $a_c$ = Fatigue limit load-life modifying factor; see Ioannides, et al. (Ref. 12)
- $\bar{a}$ = Hertzian semi-width along the rolling direction (m)
- $\bar{B}$ = Damage constant for surface
- $b$ = Hertzian semi-width across the rolling direction (m)
- $C$ = Dynamic capacity of the bearing (N)
- $c$ = Exponent in the bearing life equation
- $d_m$ = Mean diameter of the bearing (mm)
- $e$ = Exponent in the bearing life equation (standardized Weibull slope)
- $F$ = Radial bearing load (N)
- $G(N)$ = Accumulated material degradation function from 0 to $N$ load cycles
- $h$ = Thickness of the surface layer (m)
- $\bar{h}$ = Exponent in the bearing life equation
- $I_s$ = Surface damage integral or surface damage parameter
- $I_{ss}$ = Subsurface damage integral
- $K$ = Constant for the surface damage function
- $L$ = Rolling contact life, bearing life (MR ev)
- $L_{10, BR}$ = Basic rating life (MR ev)
- $m$ = Weibull slope for surface failure modes
- $N$ = Number of load cycles
- $P$ = Equivalent load in the bearing (N)
- $P_s$ = Fatigue limit load in the bearing (N)
- $p$ = Exponent in the bearing life equation
- $\Delta p$ = Pressure fluctuations due to roughness (Pa)
- $p_{max}$ = Maximum Hertzian pressure in the contact (Pa)
- $Q$ = Contact load (N)
- $R_q$ = r.m.s. value of the roughness (m)
- $R_s$ = Fatigue load limit in the bearing (N)
- $\bar{\sigma}$ = Amplitude value of stress-related fatigue criterion (Pa)
- $\sigma_w$ = Fatigue limit value of the fatigue criterion used (Pa)
- $\eta$ = Stress fatigue limit (Pa)
- $\kappa$ = Viscosity ratio in the bearing (Ref. 15)
- $\tau$ = Shear stress fatigue limit (Pa)
- $\tau_{uc}$ = Amplitude of the orthogonal shear stress (Pa)
- $\Psi_{bg}$ = Bearing-type characteristic number (Ref. 14)

**Subscripts**

- $e$ = Related to outer ring
- $i$ = Related to inner ring
- $s$ = Related to surface
- $v$ = Related to volume

**Introduction**

Since the pioneering work of Lundberg and Palmgren in 1947 (Lundberg and Palmgren (Refs. 1–2)), rolling bearing life has been modeled using basic principles of rolling contact fatigue based on an equivalent stress originated beneath the contacting surfaces and affecting the over-rolling material volume of the contact. This approach is focused on subsurface spalling fatigue because this was the dominant failure mode at the time the model was developed. In 1967, Tallian (Ref. 3) found that there are many different competing failure modes leading to fatigue failure of the bearing raceway. This is also the conclusion of a more recent investigation about
this topic performed by Olver in 2005 (Ref. 4). In 1971, Chiu, et al. (Refs. 5–6), Tallian and McCool (Ref. 7), and Tallian (Ref. 8) also attempted to tackle the different failure mechanisms occurring at the surface and in the subsurface of the rolling contact using engineering crack mechanics concepts. The different failure modes that occur in rolling bearings and their effect on rolling bearing life are extensively discussed by Zaretsky (Ref. 9).

An exhaustive and up-to-date categorization and review of rolling contact fatigue models can be found in Sadeghi, et al. (Ref. 10).

A significant contribution and further extension of the Lundberg and Palmgren (Refs. 1–2) theory is provided by Ioannides and Harris (Ref. 11) and Ioannides, et al. (Ref. 12) with the introduction of an additional material parameter to characterize the fatigue strength of bearing steel at the very high number of stress cycles (Gabelli, et al. (Ref. 13)). The new model was found to give a better representation of the load-life performance of modern rolling bearings (Ioannides, et al. (Ref. 12); Gabelli, et al. (Refs. 13–14)). Furthermore, this model provides a consistent methodology to globally derate the life of the rolling contact in case the operating conditions are less than optimal as in case of reduced lubrication or presence of contamination particles (Ioannides, et al. (Ref. 12); Gabelli, et al. (Ref. 14)). Currently this approach is well supported by national and international standards (Ref. 15).

In the last few decades, the need to further increase energy efficiency in machines and reduce their environmental impact has created the tendency for rolling bearings to operate at higher rotary speeds, higher temperatures, and reduced lubricant film thicknesses. Furthermore, the presence of contaminants and aggressive additives in the lubricant has also contributed in making surface related failures a very common aspect of current service life of rolling bearings. Because of this, there is an increased need for more versatile or generalized rolling bearing life models, able to adapt and incorporate new developed knowledge about the tribology of surface initiated failures of the rolling contact.

Despite the progress achieved in the last few years in the numerical modeling of the tribology and surface performance of rolling contacts (e.g., Epstein, et al. (Refs. 16–17); Morales-Espejel and Brizmer (Ref. 18); Morales-Espejel and Gabelli (Ref. 19); Brizmer, et al. (Ref. 20); Warhadpande and Sadeghi (Ref. 21)), the integration of this new knowledge into an engineering model for bearing life estimation is, to some extent, hindered by the simplicity of present standardized life rating formulation (Ref. 15), which only relies on averaged global de-rating factors. This approach, although sufficient for most common situations, is not designed to give an account and differentiate among surface/subsurface competing failure modes that may occur in a bearing when exposed to a hostile environment and tough operating conditions.

Objective of this Article
The objective of the present article is to describe the basic equations of a probabilistic model for rolling contact fatigue life estimation tailored to better characterize bearing surface-induced damage from the subsurface rolling contact fatigue process of the bearing. This new formulation is designed to facilitate the incorporation into bearing applications of newly developed knowledge gained from testing, advanced numerical modeling, or even engineering field experience of the expected surface and subsurface performance of the rolling contact.

The article presents the fundamental theoretical aspects of the new model and its general behavior and does not intend, at this point, to describe an engineering methodology for bearing life calculation in applications; that aspect should come in further publications. The advantages of the present method in providing a specific account for the observed surface/subsurface competing fatigue mechanisms of the rolling contact is illustrated using a simple bearing application operating under various lubrication conditions and endurance test results.

Probabilistic Damage Approach
It is well known that under laboratory conditions, seemingly identical bearings operating under identical conditions have significantly different individual bearing lives. Because of this, the prediction of bearing life requires a probabilistic setting, in order to:

1. Represent the intrinsic local variability of the material matrix strength, geometrical parameters and stochastic properties resulting from the presence of random inclusions and other inhomogeneities; for example, Ioannides and Harris (Ref. 11), Lamagnere, et al. (Ref. 22), Lai, et al. (Ref. 23), Weibull (Ref. 24).

2. Provide a simple method for the nominal scaling of the life, conventionally rated at 90% reliability; that is, \(L_{10}\) to a different value of reliability; that is, \(L_{\theta}\) (Lundberg and Palmgren (Ref. 1); ISO 281:2007 (Ref. 15); Weibull (Ref. 25)).

The present model will retain the standardized probabilistic approach used in rolling bearing life ratings based on a two parameter Weibull distribution, as discussed in Blachere and Gabelli (Ref. 26).

Weibull (Ref. 24), with the weakest link theory, introduced stochastic concepts in the determination of strength and rupture of structural elements.

If a structure is composed by \(n\) elements subjected to different stress states, thus with a different probability of survival \(S_1, S_2, \ldots, S_n\), following the product law of reliability, the probability that the whole structure will survive is:

\[
S = S_1 \cdot S_2 \cdot \ldots \cdot S_n = \prod_{i=1}^{n} S_i
\]

which can be expressed also in the equivalent form:

\[
\ln(S) = \ln(S_1) + \ln(S_2) + \ldots + \ln(S_n) = \sum_{i=1}^{n} \ln(S_i)
\]

Contact damage model. Lundberg and Palmgren (Ref. 1), in their classic original formulation of the dynamic capacity of rolling bearings in 1947, applied the product law of reliability of Weibull Equation 2 to derive the survival function of a structure made of \(n\) independent physical elements accounting for the degradation process from 0 to \(N\) load cycles:
The volume $V$ can be divided into two or more independent sources of damage risk for the structure; consider that $G$ is a material degradation function accounting for the effect of the accumulation of load cycles (fatigue). Therefore, regions can be characterized by different material degradation functions that could describe different (or a single) degradation processes, $G_{n1}$, $G_{n2}$, ..., $G_{nn}$. Their combined effect on the survival of the complete structure can be expressed by using Equation 3, from which the following can be derived:

$$\ln \left[ \frac{1}{S(N)} \right] = \ln \left[ \frac{1}{\Delta S(N)} \right] + \ln \left[ \frac{1}{\Delta S(N)} \right] + \cdots + \ln \left[ \frac{1}{\Delta S(N)} \right]$$  \hspace{1cm} (3)

The probability of survival (Eq. 6) can then be rewritten in the following form:

$$\ln \left[ \frac{1}{S(N)} \right] = \int_{V_s} G_v(N)dV_s + \hat{h} \int_{A} G_v(N)dA$$  \hspace{1cm} (6)

This formulation of the survival of a rolling contact allows the surface to account for the different probabilistic failure modes independently from the subsurface region of the contact. Indeed, surface phenomena such as surface distress, wear, indentations, frictional heating, etc., in general affect the fatigue of a very thin material layer whose mechanical properties may differ from the bulk properties of the Hertzian contact (Fig. 1). Therefore, it is advantageous to analyze the damage developed in this region separately from the rest of the contact. For instance, surface traction and roughness-induced surface stresses will not, in general, affect the subsurface smooth Hertzian stress or the amplitude of the fatigue stress criterion of the rolling contact (Ioannides, et al. (Ref. 12)).

Furthermore, in situations where surface traction is dominant, the stresses very close to the surface appear to be almost independent of the depth. Therefore, a further approximation is possible; consider a surface material volume $V_s = \hat{h} \cdot A$ as a thin layer on the raceway, then $G_v$ applies to the surface region $A$, up to a depth $\hat{h}$ of the order of the depth of the microgeometry maximum stress (Fig. 1). For rolling bearings, raceway microgeometry (Refs. 14, 27) $\hat{h}$ can be assumed small and constant for similar classes and types of bearings.

$$\ln \left[ \frac{1}{S(N)} \right] = \int_{V_s} G_v(N)dV_s + \hat{h} \int_{A} G_v(N)dA$$  \hspace{1cm} (7)

**Material degradation functions — subsurface damage.** It is well established that subsurface damage in rolling bearings is caused by rolling contact fatigue (Lundberg and Palmgren (Refs. 1–2); Sadeghi, et al. (Ref. 10)). Cumulative fatigue damage models are expressed by a stress power law to account for the portion of life spent in the initiation and the short macropropagation phase of the crack that will ultimately determine the life time of rolling contacts (Lundberg and Palmgren (Ref. 1); Weibull (Ref. 24)). Many authors believe that the power function generally used to describe fatigue processes is not a mere empirical equation, but it is recognized as having the characteristics of a general physical law for damage accumulation processes of more universal applicability (Basquin (Ref. 28); Kun, et al. (Ref. 29)). A power law for rolling contact fatigue damage can be found in Lundberg and Palmgren (Ref. 1), Ioannides and Harris (Ref. 11), and Ioannides, et al. (Ref. 12). Using the approach of Ioannides and Harris (Ref. 11), the power law for subsurface rolling contact fatigue reads:

$$G_v = \bar{c} N \left( \frac{\sigma - \sigma _0}{\sigma _m} \right)^{\eta}$$  \hspace{1cm} (8)

More advanced formulations of the same basic model are also available. In Ioannides (Ref. 30), $\sigma _0$ is not constant but it is assumed to be function of $N$; that is, $\sigma _0(N)$. A similar methodology can be adopted in case the material undergoes changes of the original fatigue strength due to some extreme thermomechanical conditions during the stress cycling history of the rolling contact. When variable operating conditions are used in a bearing, the damage accumulation can be followed up by using the Palmgren-Miner rule (Ref. 31). Therefore, when the fatigue limit changes due to an extreme event — for example,
overloading, overheating etc.—Equation 8 can still be used with the current, derated fatigue limit.

**Material degradation functions—surface damage.** Several surface-related failure modes and related mechanisms can be identified in rolling bearings (e.g., surface distress, indentations, wear-related stress concentrations, micropitting, surface chemistry, etc.). Under severe operating environmental conditions, surface damage leads generally to failures that are quite independent from the subsurface fatigue strength. Contrary to that, surface survival is more related to the operating conditions and raceway micrometry e.g., metal-to-metal contact, local friction, film thickness, etc.). It is therefore difficult to generalize all different mechanisms using a single damage function as proposed for the subsurface fatigue case. Specific damage functions should be related to the expected failure modes. Tribological models can be used to derive these damage functions. When several mechanisms are present—for example, surface distress and mild wear—the damage function should account for possible competitive mechanisms (Ref. 18) and the statistical treatment should follow (Ref. 32). As an example of surface fatigue (Ref. 33), the following damage function is considered:

\[ G_s = B N_0 (\sigma - \sigma_c)^m \]  

Generalizing Equation 7 for various independent surface regions (flanges, raceways, etc.) and/or independent surface damage mechanisms (surface distress, surface chemistry, etc.), one has:

\[ \ln \left[ \frac{1}{N(N_0)} \right] = \int_V G_s(N) dV + \int_A G_s(N) dA + ... + \int_A G_s(N) dA \]  

Note that to facilitate the use of the Weibull statistics (Weibull (Ref. 25)), the survival Equation 10 would require the adoption of a common standardized Weibull shape parameter (slope) for all different degradation functions. Failure modes with a tendency to be deterministic (very large Weibull slope) may represent a problem when combined with the more probabilistic classical rolling contact fatigue; for those cases (e.g., severe smearing, very severe wear, or contamination), the current approach may show limitations.

**Model Behavior**

Following Ioannides and Harris (Ref. 11) and Equation 8, the fatigue damage volume integral can be obtained by using the stress amplitude \( s \) originated from the Hertzian stress field:

\[ \int_V G_s(N) dV = B N_0 \int_V \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dV \]  

In a similar manner one can rewrite the surface damage function. If the constant \( \bar{B} \) is included into the surface damage proportionality constant \( B \), one obtains:

\[ \frac{\bar{B}}{N_0} \int_A G_s(N) dA = \bar{B} N_0 \int_A \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dA \]  

In the surface damage function (Eq. 12) the stresses \( \sigma \) must be obtained from the actual surface geometry of the contact and frictional stresses. Furthermore, for the sake of generality, the fatigue failure distribution of the surface of the rolling contact will be allowed to have a different Weibull slope \( m \) compared to the fatigue failures distribution of the subsurface volume \( e \). If different Weibull slopes are introduced when combining surface and subsurface damage models, the resulting statistics will not follow a standard Weibull failure distribution model. Substituting in Equation 7 and rearranging gives:

\[ \ln \left( \frac{1}{N} \right) = N \left[ \frac{1}{\bar{B}} \int_V \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dV + \frac{N}{m - e} \int_A \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dA \right] \]  

Substituting \( N = u L \) in Equation 13 and solving leads to:

\[ \frac{1}{L^e} = \frac{1}{L^u} - \frac{1}{L^e} \frac{u \sigma_c}{\bar{B}} \ln \left( \frac{1}{N} \right) \int_A \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dA \]  

Performing the reciprocal of Equation 14 provides the following:

\[ \frac{1}{L^u} - \frac{1}{L^e} = \frac{u \sigma_c}{\bar{B}} \ln \left( \frac{1}{N} \right) \int_A \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dA \]  

It is easily recognized that in Equation 15 the reciprocal of the first term (at the right side of the equation) corresponds to the original Lundberg-Palmgren (Ref. 1) model (basic rating) modified with the additional effect of the fatigue limit \( L_e \) as introduced in Ioannides and Harris (Ref. 11), and the reciprocal of the second term (at the right side of the equation) corresponds to any additional effect introduced by damage accumulation at the surface of the rolling contact \( L_u \).

From Equation 14 it is possible also to derive an expression of the rolling contact life:

\[ L = \left[ \frac{\ln \left( \frac{1}{N} \right) \int_A \left( \frac{\sigma - \sigma_c}{\sigma_c} \right)^m dA}{u N_0} \right]^{-1} \]  

Notice that in order to obtain the calculated life from Equation 16, a calculation based on an iteration scheme is required because the rolling contact life is also included in the right-hand side of Equation 16. However, if \( m = e \), then the \( (uL)^m \) term reduces to 1 and the solution for the life \( L \) becomes fully explicit.

**Surface model behavior.** In the current formulation, the treatment of the surface stresses and damage can be accomplished by using an advanced surface distress model for elasto-hydrodynamically lubricated (EHL) rolling-sliding rough contacts developed by Morales-Espejel and Brizmer (Ref. 18); further description of this model is given below.

In the current section, the use of this model will be limited to the calculation of the stress terms in the integrals of Equation 11 for the subsurface and Equation 12 for the surface. As a simple indication, consider maximum values of the stress amplitudes. For the case of \( m = e \) the ratio of these stress amplitude terms should be proportional to the life ratio surface/subsurface to the power \( e/c \), from which a consistency check and verification of the behavior of the present model can be obtained. With the model, it is possible to calculate the local pressures and stresses from mixed-lubrication conditions using as input a 3-D digital map of the surface topographies
of the two rolling contact surfaces. The model can also calculate the local surface tractions from the mixed lubrication conditions.

This model was applied to a sample area of the raceway of a single-contact case of a radial ball bearing 6217 (Fig. 2).

In general, this type of calculation is performed on a statistical significant number of area samples. However, for the present demonstration case, one area sample was found sufficient. The calculations were performed using the set of load cases as given in Table 1 and the roughness topography sample displayed in Figure 2. In Figures 3 and 4 some examples of the numerical calculations are shown (at the maximum pressure point in time); the sample area is at the center of the rolling contact; load case \( P_u/P = 0.5 \) and with the assumption of reduced bearing clearance it is found a maximum contact pressure of \( p_u = 1.66 \text{ GPA} \) in the heaviest loaded contact (used as mean pressure in the micro-EHL analysis). The Hertzian contact semi-widths for this case are 0.24 and 2.4 mm. At this point it is important to introduce a common bearing-related term to describe the lubrication quality of the bearing, named \( \kappa \), defined as the viscosity ratio between the actual used lubricant viscosity in the bearing and the required viscosity for proper lubrication (Ref. 15). This viscosity ratio can be related to the commonly used \( A \) parameter in the lubrication of other machine elements, as discussed in (Ref. 27). At the top of Figures 3 and 4 are displayed the distribution of the surface traction resulting from the frictional condition of the contact. In the poor lubrication case, surface traction values are substantially higher as boundary lubrication conditions dominate. In contrast, for the full-film case, values are low and there is no boundary (or dry) contact at all. At the center of Figures 3 and 4 the contact pressure and the \( \tau_{xz} \) shear stress distribution for \( y=0 \) are also shown. Finally, at the bottom of Figures 3 and 4 the map of \( \tau_{xz} \) at the surface of the sample area is displayed. Also in the case of the shear stress, the poor lubrication condition provides significantly higher stress values compared to the full-film conditions.

In Table 1 the results of several calculations are summarized. The calculations were performed using the advanced numerical model for surface distress discussed earlier (Ref. 18), the surface roughness sample was subjected to four loads and two lubrication conditions; that is, lubrication conditions, high film (\( \kappa = 4 \)) and low film (\( \kappa = 0.1 \)). Furthermore, considering the manufacturing machining of the surface and other chemical absorption processes of the material, it can be safely assumed that \( \tau_{xz} = 0 \) (conservative calculation).

Table 1 shows that at very high loads (i.e., low \( P_u/P \) values) the subsurface volume stress is dominant, providing a very low value of surface over subsurface damage ratio. As the load is reduced, the surface stress effect becomes more significant in defining the bearing performance. This is indicated in Table 1 by the continuous increase in the life ratio for both poor and good lubrication conditions. Comparing the two cases, it is found, as expected, that in the poor lubrication case the ratio surface over subsurface damage is always significantly larger than in the case of high kappa conditions. The maximum difference of the life ratio between the two lubrication conditions is about of three orders of magnitude, which is consistent with the expected variation of bearing life found in today’s rating standards for the kappa range \( 0.1 < \kappa < 4 \).

### Table 1: Comparative surface and subsurface stress ratios for the example of the 6217 bearing, stress amplitude (\( \text{GPa} \))

<table>
<thead>
<tr>
<th>( P_u/P )</th>
<th>( \rho_u ) (( \text{GPa} ))</th>
<th>( \tau_{xz} - \tau_{u} )</th>
<th>( \tau_{xz,s} - \tau_{u,s} )</th>
<th>( \tau_{xz,s} - \tau_{u,s} ) ( \kappa = 0.1 )</th>
<th>( \tau_{xz,s} - \tau_{u,s} ) ( \kappa = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>7.74</td>
<td>1.7 \times 10^{6}</td>
<td>0.43 \times 10^{6}</td>
<td>2.806 \times 10^{6}</td>
<td>0.32 \times 10^{6}</td>
</tr>
<tr>
<td>0.05</td>
<td>3.59</td>
<td>0.599 \times 10^{7}</td>
<td>0.45 \times 10^{6}</td>
<td>6.99 \times 10^{7}</td>
<td>0.14 \times 10^{9}</td>
</tr>
<tr>
<td>0.2</td>
<td>2.26</td>
<td>0.25 \times 10^{8}</td>
<td>0.47 \times 10^{7}</td>
<td>3.54 \times 10^{3}</td>
<td>0.125</td>
</tr>
<tr>
<td>0.5</td>
<td>1.66</td>
<td>0.09 \times 10^{9}</td>
<td>0.45 \times 10^{8}</td>
<td>3.16 \times 10^{6}</td>
<td>0.14 \times 10^{9}</td>
</tr>
</tbody>
</table>

**Figure 2** Inner ring sample roughness of radial ball bearing 6217, as used in the calculations; the ball surface was assumed smooth.
Figure 3 Calculated surface traction (along rolling direction), pressure, and stress $\tau_{xz}$ profile at $y = 0$; surface stress $\tau_{xz}$ at over-rolling position of maximum mean pressure; low kappa case, $\kappa = 0.1$; rolling direction from left to right.

Figure 4 Calculated surface traction (along rolling direction), pressure, and stress $\tau_{xz}$ profile at $y = 0$; surface stress $\tau_{xz}$ at over-rolling position of maximum mean pressure; high kappa case, $\kappa = 4$; rolling direction from left to right.
**Bearing Model Formulation**

The transformation of the single-contact model Equation 16 into a rolling bearing life equation can be done by following the work of Lundberg and Palmgren (Ref. 1). Starting from Equations 44 and 46 of Lundberg and Palmgren (Ref. 1), after some algebraic manipulation to account for the load distribution and internal geometry in a bearing, Equation 81 of this reference gives the transformation into bearing load. For a bearing load \( F \) applied radially on the rolling bearing, the heaviest loaded contact rolling-element (inner ring, \( i \) ) and for the contact rolling-element (outer ring, \( j \) ), one has:

\[
\begin{align*}
\ln(1/S_j) &= k_m^{w \cdot F'_{x} L'} \\
\ln(1/S_i) &= k_m^{w \cdot F'_{x} L'}
\end{align*}
\]

where the contact load of the heaviest loaded rolling-element \( Q \) and the bearing load \( F \) are related by \( F = m_i Q \) and \( F = m_j Q \), and the exponent \( w = p/e = (c - h + 2)/3 \). The parameters \( m_i, m_j \) are internal geometry functions of the bearing and can be calculated from Equations 89 and 95 of (Ref. 1). Finally, \( k_i \) and \( k_j \) are proportionality constants defined according to the reference and with the use of the nomenclature given in the reference; thus:

For point contact —

\[
k_i = \ln(1/S_i)(A_{g} \cdot \phi \cdot \sigma^{c - 1})D_{v} e^{(-2c + h - 3)}
\]

For line contact —

\[
k_j = \ln(1/S_j)(B_{g} \cdot \phi \cdot \sigma^{c - 1})D_{v} e^{(-2c + h - 3)}
\]

Combining the probabilities \( S = S_i S_j \), leads to:

\[
\ln(1/S) = (k_m^{w \cdot F^{x} L^{x}})
\]

Solving for \( L \) one obtains:

\[
L = \left[ \frac{\ln(1/S)}{k_m^{w \cdot F^{x} L^{x}}} \right]^{1/3} \frac{1}{P^{x}}
\]

Now, following Ioannides, et al. (Ref. 12; Eq. 8), the damage volume integral can be obtained by a mean approximated value (avoiding in this way the integration):

\[
\int_{s} G_{s}(N) dV = \bar{A} N \left( \frac{\sigma - \sigma_{y}}{\bar{e}_{s}} \right) ab
\]

For the surface the stress integral can be derived from Equation 12 as:

\[
\hat{k} \int_{s} G_{s}(N) dA = \bar{B} N^{w} \int_{s} (\sigma_{x} - \sigma_{y}) dA = N^{w} I_{s}
\]

where, \( I_{s} \) represents the unknown surface damage integral, which includes a constant layer thickness \( \hat{k} \) in the constant \( \bar{B} \).

Substituting in Equation 7 and rearranging gives:

\[
\ln(\frac{1}{S}) - N^{w} I_{s} = \bar{A} N^{w} \left( \frac{\sigma - \sigma_{y}}{\bar{e}_{s}} \right) ab
\]

From Equation 24 it follows that Equation 21 can be modified with a surface integral by replacing \( \ln(\frac{1}{S}) \) by \( \ln(\frac{1}{S}) - N^{w} I_{s} \), so replacing \( F \) with the more general bearing notation, \( P \), and introducing a fatigue limit load-life modifying factor (Ref. 12), here it will be named \( a_{v} \). Notice that this function becomes \( A \) in case of zero fatigue limit.

\[
a_{v} = \frac{\bar{A}}{1 - \left( \psi_{m}^{B/P} \right)^{\psi/v}}
\]

Therefore, following Equation 21:

\[
L' = \left[ \ln(1/S) - N^{w} I_{s} \right] (a_{v})\left( \frac{1}{P^{x}} \right)
\]

and with \( N^{w} = u^{w} L^{x} \):

\[
L' = \left[ \ln(1/S) - u^{w} L^{x} \right] (a_{v})\left( \frac{1}{P^{x}} \right)
\]

Similarly, with existing bearing life equations the square bracket in Equation 27 without the \( I \) term and with \( S = 0.9 \) represents the dynamic capacity of the bearing to the power \( P \); thus:

\[
L_{10} = \left[ \frac{1}{1 - u^{w} L^{x} I_{s} \ln(1/0.9)} \right] \left( \frac{C}{P} \right)^{x} \left( a_{v} \right)\left( \frac{C}{P} \right)^{x}
\]

Finally, solving for \( L_{10} \) gives:

\[
L_{10} = \left[ \frac{1}{1 - u^{w} L^{x} I_{s} \ln(1/0.9)} \right] \left( \frac{C}{P} \right)^{x} \left( a_{v} \right)\left( \frac{C}{P} \right)^{x}
\]

Equation 29 represents a new general semi-analytical form for bearing life calculation, based only on the modifying factor \( a_{v} \) with no surface damage components. It introduces an additional new surface damage function (or integral) \( I \), to account for only the surface effects in bearing life, calculated from Equation 23 or back-calculated with the use of an advanced surface distress models.

**Advanced surface distress model.** An advanced surface distress model (fatigue and mild wear) was introduced in the past (Ref. 18); this model has been validated with experiments and has been used with success in the description of indentation failures in rolling contacts (Ref. 19). The model will not be described here in detail as it is described elsewhere, but for completeness of the current article the key elements of the model are described below; the model will be used later on to illustrate the potential of the current formulation.

Modeling of the surface-fluid interactions (pressure and surface shear stress) is carried out by using a mixed-lubrication model that solves the transient Reynolds equation (with non-Newtonian behavior) for the fluid part and

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**Figure 5** Illustration of roughness relative movement inside the contact; local analysis of the topography is done by varying the mean pressure following to local Hertz value (Ref. 18).
the half-space elasticity for the solid component with a fast Fourier transform technique. For every macrocycle (Fig. 5), the scheme simulates the relative movement in time of the microgeometry inside the contact due to sliding (including moving pressures and stresses) and accumulates the damage via a fatigue criterion. Because at every passage in the contact the local operating conditions and the topography itself may change; thus, a simple damage accumulation law is used. Figure 6 summarizes the general modeling scheme and the computer program flowchart. The process begins with measured virgin 3-D surface samples for the two contacting bodies. Once both rough surfaces enter into the contact in the first cycle, the transient mixed-elastohydrodynamic lubrication problem is solved by means of the Reynolds equation for the different time steps until the roughness samples exit the contact. In the neighborhood of the surface the full stress tensor is calculated and used in the fatigue damage criterion (Ref. 18), which is applied for the current cycle, then the damage is accumulated according to an accumulation law (i.e., Palmgren-Miner) for the calculated number of cycles. Next, if the accumulated damage parameter exceeds the crack initiation limit, a crack or a micropit will be generated. The surface distress for the different time steps until the roughness samples exit the contact. In the neighborhood of the surface the full stress tensor is calculated and used in the fatigue damage criterion (Ref. 18), which is applied for the current cycle, then the damage is accumulated according to an accumulation law (i.e., Palmgren-Miner) for the calculated number of cycles. Next, if the accumulated damage parameter exceeds the crack initiation limit, a crack or a micropit will be generated. The surface distress (within 1.5% damage area at the surface of the total simulated area) could be calculated from the following curve-fitted equation:

$$N_{1.5\%} = \exp \left\{ \frac{1}{\log_{10} c_1} \left[ \frac{c_2}{\mu} - \left( c_4 P_c \right)^{c_3} + c_4 \right] \right\} \tag{30}$$

where, $c_1$, $c_2$, $c_3$, and $c_4$ are curve-fit constants that depend on the lubrication quality parameter of the bearing $\kappa$ and the bearing mean diameter $d_m$.

The core of Equation 30 can be rearranged in terms of bearing load ratio rather than pressure and simplified to represent a surface damage function; therefore:

$$u^* I = f_1 \exp \left\{ \frac{f_2}{(P/P_u)^{c_1}} + \frac{f_3}{(P/P_0)^{c_2}} \right\} \tag{31}$$

where, $f_1$, $f_2$, $f_3$, and $f_4$ are curve-fitted constants that depend on the surface stress conditions (e.g., lubrication, contamination, etc.).

Finally, Equation 31 can be solved for $f$’s using some calculated points of $u^* I$, obtained with the advanced surface distress model with the use of a collocation algorithm by fixing a number of locations in the abscissa $P/P_u$. An example of the obtained surface fatigue function for no contamination conditions is shown (Fig. 7) ($R_s$) normalized with respect to a constant; it can be seen that for better lubrication conditions (higher $\kappa$ values), the surface fatigue function is substantially reduced, and it is also nearly constant with load.
reduced. This function is also nearly constant with load at high values of load.

**Ratio of surface damage.** To further explore the consistency of the new approach, the ratio between the surface and subsurface damage functions is calculated and discussed. Now, with introduction of the definition of the basic rating life (e.g. Ref. 15) $L_{10, BR} = (C/P)^{p}$, this ratio is calculated by:

$$S_{R} = \frac{(a_{uL_{10, BR}} - L_{10})}{(a_{uL_{10, BR}})}$$  \hspace{1cm} (32)

In this equation $L_{10}$ is calculated with the use of Equation 29 and the proposed surface damage diagram (Fig. 7).

Figure 8 shows the corresponding plots for $S_{R}$ as a function of load $P/P_{u}$ and $\kappa$. From the figure it can be observed that at very low loads the surface damage tends to low values; with an increase in load the surface damage becomes dominant with respect to the subsurface damage. However, with even higher loads the subsurface damage gains importance due to rolling contact fatigue, reducing the dominance of the surface damage.

Figure 8 also shows an approximate indication of when the damage is driven by the surface and when by the subsurface, depending on the load and the lubrication conditions, giving this a possible indication of where most likely the failure will occur at the end of the life of the bearing population. It can also be seen that the importance of the surface damage function with respect to the subsurface is reduced when the lubrication conditions are enhanced (higher $\kappa$ values).

**Results and Discussion**

In the present section, some results from the model will be compared with endurance tests carried out in-house; furthermore, a simple ball bearing example will be used to illustrate the methodology. The comparison with endurance tests will focus on the surface model, which is the novel part of the present approach. The subsurface model used here has been compared with endurance tests in the past (e.g., Refs. 11–12) and so will not be repeated here.

Endurance testing practice (Refs. 34–35) shows that the rate
of bearing failure, either from the surface or from the subsurface (Fig. 9), is distributed in a very similar way (Ref. 34). This indicates that a common Weibull slope can be used in Equation 29. This equation can be used to back-calculate the surface damage parameter \( R_s = u^L_e/[Kln(1/0.9)] \) corresponding to 90% reliability of a bearing populations that are endurance tested.

The methodology used is as follows:
1. Solve Equation 29 for \( I \), with known operating conditions and \( L_{10} \) lives from endurance tests.
2. Calculate the parameter \( R_s \) using \( L_e \).
3. Repeat the calculation for every endurance test series and plot the results \( R_s \) vs. \( P/P_r \) and \( \eta \) in Figure 10 (points in the plot). As described in Gabelli, et al. (Ref. 14), the parameter \( \eta \) is a measure of the stress on the surface caused by poor lubrication and/or contamination conditions; for \( \eta = 1 \) there is no extra surface stress, for \( \eta = 0 \) the stress is maximum (Ref. 12).
4. Once the test series are plotted, plot the results of the surface distress model (solid lines) (Eq. 31). The dependence on \( \eta \) of this equation is implicit in the surface topographies and operating conditions used to obtain this equation. In this case, the extreme conditions of the tests have been used.

Typically a bearing population sample is formed by a set of 25–35 bearings, of which about one third may fail during the test (Ref. 36). For this evaluation, a set of 227 endurance population samples was used including a total of some 6,650 bearings, covering both ball and roller bearing geometries in equal proportion. A detailed description of the test used is given in (Ref. 14), but for the sake of completeness a summary of the test conditions is given in Table 3. From the test, the mean point estimation of the \( L_{10} \) life is derived using Weibull statistics. In Figure 10 the surface damage parameter \( R_s \) back-calculated from a large set of endurance tests results (dots) is displayed alongside the surface damage parameter obtained from the surface distress model presented in Equation 31 (lines). The curves of the surface distress model represent the limit conditions of the endurance tests for both roller and ball bearings and are plotted vs. the surface stress concentration reduction factor \( \eta \) (Ref. 14) for description. Inspecting the results plotted in Figure 10, it is apparent that almost all test results are positioned below the limit curves obtained from Equation 31. From Figure 10 it can be concluded that the model theory provides a safe estimation of the raceway survival. Furthermore, given the large dispersion affecting endurance test results, an additional experimental evaluation was carried out. A more recent group of endurance test results was merged into a single pool of results formed by 445 roller and ball bearings tested under reduced lubrication \((k = 0.4–0.5)\). This test pool was statistically analyzed in order to gain information regarding the 90% confidence interval of the surface damage parameter derived from testing. The result of this analysis is indicated with the square symbol and the 90% confidence range is indicated with the error bars. The comparison with the model curves shows that also in this case the surface damage function has a proper safe setting compared to the statistical data of the survival of the raceway surface. The test series (points) follows very well the behavior of the surface model when the severity at the surface \( (\eta) \) is varied; higher values of \( \eta \) represent endurance tests with good lubrication and little or no contamination (less severity at the surface), and the surface model (lines) shows the corresponding behavior.

A ball bearing example. Despite the fact that the objective of the present article is not to illustrate the use of the proposed model in bearing applications nor in comparison with other models, it is believed that a simple ball bearing example can help the reader to understand the methodology and the behavior of the model.

Consider a standard 6309 deep-groove ball bearing with a dynamic load rating of 55.3 kN and a fatigue load limit of 1.34 kN, working under a radial gravity load of 10 kN. The bearing is operating under constant lubrication and temperature conditions in different rotating speeds; thus, the lubrication parameter \( k \) described in ISO 281:2007 (Ref. 15) can be readily calculated. The bearing works under very clean conditions so, the contamination parameter (see ISO 281:2007 (Ref. 15)) can be set \( e = 1 \).

Using the surface–subsurface rating life calculation method outlined in the present article, the rating life can be estimated using Equation 29 in conjunction with Equation 25 and Figure 7. For this calculation example, the surface–subsurface life rating life model can be set using similar constants and parameters as applied in the ISO 281 model (Ref. 15) and thereby similar results from the two models are expected.

Following this criterion it is assumed that the surface/subsurface survival distribution is characterized by similar, standardized Weibull exponents; thus the surface Weibull exponent \( m \) can be assumed to be equal to the Weibull exponent \( e = 10/9 \) as used in ISO 281:2007 (Ref. 15). Others exponents and constants used in Equations 29 and 25 can be taken from (Ref. 12), leading to the results of Table 4.

<table>
<thead>
<tr>
<th>Bearing Type</th>
<th>Designation</th>
<th>Load Ratio, C/P</th>
<th>Lubrication, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep-Groove Ball Bearings</td>
<td>6305, 6205, 6206, 6207, 6309, 6220</td>
<td>1, 2.8, 2.4, 2.1, 3.1, 3.5, 4, 6</td>
<td>4, 3.4, 2.1, 2.1</td>
</tr>
<tr>
<td>Cylindrical roller bearings</td>
<td>NU 207 E, NU 309 E</td>
<td>2.5, 2.77, 2.82</td>
<td>4, 1.0, 8</td>
</tr>
<tr>
<td>Spherical roller bearings</td>
<td>22220 E, 22220 CC</td>
<td>2.22, 2.3, 2.5, 2.7, 3, 4.7</td>
<td>4, 3.6, 1.8, 0.37, 0.28</td>
</tr>
<tr>
<td>Tapered roller bearings</td>
<td>331274, K-LM11749, K-HM89449, K-580/572</td>
<td>1, 1.1, 1.3, 2.5, 3.5</td>
<td>4, 2.9, 0.9</td>
</tr>
</tbody>
</table>

By inspecting Table 4 it can be recognized that the surface/subsurface life model (when positioned to the same settings...
as in ISO 281 (Ref. 15)) can provide rating lives that are similar to the standard.

However, the main ability of the present model is to treat fatigue damage developed at the surface of the raceway separately from the subsurface. This is the motivation behind the development of the present model, as this will open new and better capabilities in representing the performance and damage process taking place in rolling bearings.

**Summary and Conclusions**

New concepts for bearing life calculation based on the separation of different regions at risk have been investigated. A simple approach is proposed to separate the surface from the subsurface rolling contact fatigue based on the product law of reliability. In this way a more flexible and physical approach can be constructed to describe surface damage mechanisms in rolling bearings considering tribological effects on the surfaces. As an example of the potential of the new approach consisting of the separation of surface and subsurface, the effects of poor lubrication were considered in the current article and an advanced model for surface distress (Ref. 18) was used to describe these effects in a more general bearing life calculation. With similar techniques, it seems possible to include, for example, the effect of additives (Ref. 20), indentation (Ref. 19) and abrasive wear — to be discussed in future publications.

From the development of the model and its behavior, the following observations can be summarized:

- There are significant gains and increase in flexibility in bearing life modeling when other failure modes or regions in addition to the Hertzian rolling contact fatigue are incorporated into the formulation of the bearing life.
- The present approach represents a framework where different failure modes can be included for different regions at risk of the bearing contact. This approach allows for the incorporation of knowledge gained from the use of tribological models into bearing life estimation.
- The current approach can give an indication of the zone at higher fatigue risk in a bearing population.
- A potential target of the new approach is the physical modeling of several classes of bearing failure mechanisms, such as surface distress (surface fatigue and mild wear), lubricant contamination, lubricant additives, wear-related stress concentrations, etc., capturing some deterministic aspects of bearing surface topography, material properties, and local lubrication conditions.

A new more general model able to account for different failure modes has been proposed, and the surface term of the model is compared with a large set of experimental results of endurance testing of bearing population samples. With the use of advanced tribological models, curve-fitted equations are obtained to describe the surface damage functions $L_i$ to consider; for example, surface distress effects from poor lubrication. Furthermore, a simple application example of a ball bearing has been used to illustrate the methodology.

From the obtained results the following conclusions can be drawn:

- In the present work, the separation of the surface and subsurface in bearing life calculations has been demonstrated to be a feasible and convenient way to model particular failure modes in the two separate regions.
- The proposed surface damage parameter $R$, obtained from the advanced surface distress model represents a good conservative limit when compared with endurance tests results.
- According to the results of Table 1 and the ball bearing example, the subsurface and the surface fatigue models keep a consistent behavior when compared with existing bearing life models. **PTE**

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**References**


**Antonio Gabelli** is senior scientist working at SKF on tribology and rolling contact fatigue life of rolling bearings. He joined the Engineering and Research Center of SKF in Netherlands in 1981, since than he has published many research papers on fatigue and lubrication of rolling bearings. He was one of the key contributors to the new bearing life theory, developed by SKF in the 90’s He is also author of several patents on rolling bearings. He holds a mechanical engineering degree from the University of Padua (Italy) and a PhD degree in from Chalmers Technical University of Gothenburg (Sweden).

**Guillermo E. Morales-Espejel** is principal scientist, SKF Engineering & Research Centre, The Netherlands, and a visiting professor at LaMCoS, INSA de Lyon. He holds a PhD in tribology from the University of Cambridge, a “Habilitation à Diriger des Recherches (INSA-Lyon)” and has 15 years of experience in rolling bearings. Morales-Espejel has authored more than 50 scientific papers and several book chapters with a focus on bearing life, friction, lubrication and surface life.

**Antonio & Guillermo** were awarded by STLE for their tribology publications of 2010 (“Particle Damage in Hertzian Contacts and Life Ratings of Rolling Bearings”) & 2013 (“The Behaviour of Indentation Marks in Rolling-Sliding Elasto-hydrodynamically Lubricated Contacts.”)