Rolling Bearing Service Life Based on Probable Cause for Removal — A Tutorial

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From the Author…
With improved manufacturing and steel processing, together with advanced lubrication technology, the potential improvements in bearing life can be as much as 80 times that attainable in the late 1950s, or as much as 400 times that attainable in 1940. Today, bearing fatigue probably accounts for less than 5% of all bearings removed from service for cause. Of ~224,000 commercial aircraft engine bearings removed from service for rework, 1,977 or 0.88% were rejected because of fatigue.

What's new?
A bearing service life prediction methodology and tutorial indexed to eight probable causes for bearing failure and removal are presented — including fatigue. Bearing life is probabilistic and not deterministic. Bearing manufacturers' catalogue (L_{10}) bearing life is based on rolling-element fatigue failure, at which time 90% of a population of bearings can be reasonably expected to survive, and 10% to fail by fatigue. However, approximately 95% of all bearings are removed for cause before reaching their L_{10} life. A bearing failure can be defined as when the bearing is no longer fit for its intended purpose. For a single bearing, you can only predict the probability of a failure occurring at a designated time — but not the actual time to failure.

We — and the author — want to know what you think about the bearing service life methodology and tutorial presented in this paper. Especially if you are manufacturing, buying or selling bearings in great quantities — and you have a question or comment regarding how this alternative methodology might affect your business — please send your questions or comments to jmcguinn@powertransmission.com.

Nomenclature

F = Probability of failure (fractional percentage or percentage)
F_n = Probability of failure of a chain consisting of n links (fractional percentage or percentage)
L = Life, cycles (stress cycles); inner- or outer-ring revolutions (h)
L_{ref} = Reference life, inner or outer ring revolutions (h)
L_{serv} = Bearing service life, inner- or outer-ring revolutions (h)
L_β = Characteristic life (time at which 63.2% of a population will fail, or 36.8% will survive), cycles (stress cycles), inner- or outer-ring revolutions (h)
L_μ = Location parameter or time below which no fatigue failure should occur, cycles (stress cycles), inner- or outer-ring revolutions (h)
m = Slope of the Weibull plot or Weibull modulus
n = Number of independent components
P = Load (N or lbs.)
S = Probability of survival
X = Number of bearings removed from service because of fatigue divided by all bearings removed from service regardless of cause (fractional percentage)

Introduction

In the first edition of his book, *Ball and Roller Bearing Engineering*, Dr. Arvid Palmgren (Ref. 1) defines the term (bearing) life as follows:

> No bearing gives an unlimited length of service. If a ball or roller bearing is exposed to moisture or dirt, it may be rendered unserviceable due to rust (corrosion) or wear, after a period of service which obviously cannot be predicted. However, if it is effectively protected, well lubricated, and otherwise properly handled, all causes of damage are eliminated except one, the (rolling-element) fatigue of the material due to repeated stresses under rotation. The effect of this fatigue is the so-called flaking, which starts as a crack and develops into a spalled area on one or the other of the load carrying surfaces. Fatigue is, ultimately, unavoidable but the number of revolutions the bearing may make before flaking starts is a function of the bearing load. The term "LIFE" can therefore be given a more exact definition to mean that period of performance which is limited by (rolling-element) fatigue phenomena. Life is measured in number of revolutions of the bearing or the number of hours of operation at a certain speed of rotation. Individual bearings which are apparently identical and which operate under identical conditions may, however, have different lives (p 68).

The L_{10} life, or the time that 90% of a group of bearings will exceed without failing by rolling-element fatigue, is the basis for calculating bearing life and reliability today. Accepting this criterion means that the bearing user is willing in principle to accept that 10% of a bearing group will fail before this time and 90% will survive.

The rationale for using the L_{10} life was first laid down by Palmgren in 1924 (Palmgren (Ref. 2)). He states:

The (material) constant C has been determined on the basis of a very great number of tests run under different types of loads. However, certain difficulties are involved in the determination of this constant as a result of service life demonstrated by the different configurations of the same bearing type under equal test conditions. Therefore, it is necessary to state whether an expression is desired for the minimum, (for the) maximum, or for an intermediate service life between these two extremes. In order to obtain a good, cost-effective

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result, it is necessary to accept that a certain small number of bearings will have a shorter service life than the calculated lifetime, and therefore the constants must be calculated so that 90 percent of all the bearings have a service life longer than that stated in the formula. The calculation procedure must be considered entirely satisfactory from both an engineering and a business point of view, if we are to keep in mind that the mean service life is much longer than the calculated service life and that those bearings that have a shorter life actually only require repairs by replacement of the part which is damaged first (pp 5–6).

Palmgren is perhaps the first person to advocate a probabilistic approach to engineering design and reliability. Certainly, at that time, engineering practice dictated a deterministic approach to component design. This approach by Palmgren was decades ahead of its time. What he advocated is designing for finite life and reliability at an acceptable risk (Zaretsky (Ref. 3)).

By the close of the 19th century, the rolling-element bearing industry began to focus on sizing of ball and roller bearings for specific applications and determining bearing life and reliability. However, before the 1924 work of Palmgren (Ref. 2), it would appear that rolling-element bearing fatigue testing was the only way to determine or predict the minimum or average life of ball and roller bearings. In 1896, Professor Richard Stribeck (Ref. 4) in Germany began fatigue testing full-scale rolling-element bearings. In 1912, Professor John Goodman (Ref. 5) in Great Britain published formulae based on fatigue experiments that he began in 1896 to compute safe loads on ball and cylindrical roller bearings (Zaretsky (Ref. 6)).

To the best of the authors' knowledge, a database that defines and/or determines the life and reliability of rolling-element bearings at the beginning of the 20th century is not readily available. In 1914, the *American Machinists Handbook* (Colvin and Stanley (Ref. 7)) devoted six pages to rolling-element bearings that discussed bearing sizes and dimensions, recommended (maximum) loading, and specified speeds. However, the publication did not address the issue of bearing life. Nevertheless, the qualitative lives of these bearings can be inferred from Stribeck (Ref. 4), wherein Henry Hess translated Stribeck's work from German to English, which was published in the 1907 *Transactions of the American Society of Mechanical Engineers*. Thomas J. Fay (Stribeck (Ref. 4)) wrote a discussion to Hess's presentation wherein he states as follows:

> The life of a ball bearing is dependent upon numerous considerations of design and upon the sizes used and the mode of application; but tests now under way in the establishment represented by the writer (Mr. Fay's affiliation is not given) indicate that trouble can be expected well within 20,000 car miles from all but the finest products, even if the load is one-half the catalogue ratings. Of course plain bearings would fail long before this under the same load conditions. But the very best of ball bearings using the most appropriate grades of steel should survive 50,000 car miles (p 464).

In his reply to Fay's discussion, Hess (Stribeck (Ref. 4)) states as follows:

> Changes in design and fashion of automobiles are such as to make the amortization life certainly not over five years, so that their bearings should not require renewal inside of that time. Few cars will average 50 miles per day for 250 days per year or a total of 62,500 miles. I have in my possession bearings taken from a heavy touring car that has been roughly used in racing and hard driving; these, with a known record of 65,000 miles, show no evidence of deterioration. Other records on standard passenger steam railways are over 200,000 miles with no visible effect on the bearings (p 466).

If we can assume a 1907 automobile tire diameter of 30 in. (76.2 cm), we can calculate the number of bearing revolutions for 65,000 miles of operation. This would suggest a life approximately equal to 43,719,745 bearing outer-ring revolutions for an automotive wheel bearing application at that time. If we further assume that the average speed of a 1907 automobile was 25 mph, the life of the bearing would be approximately 2,600 h. Based on 20,000 miles of operation the bearing life would be 800 h. Accordingly, it can be reasonably assumed that in 1907, bearing lives ranged from less than 800 h to as much or greater than 2,600 h at outer-ring speeds of 280 rpm. In terms of current bearing lives, these times are relatively low.

In 1910, A.-B. Svenska Kullager-Fabriken (SKF) bearing company in Sweden began rolling-element bearing endurance testing (Styri (Ref. 8)). These bearing fatigue tests became the basis of Palmgren's 1924 published bearing life analysis (Palmgren (Ref. 2)). In 1939, W. Weibull (Refs. 9–10), also of Sweden, published his theory of failure and the Weibull distribution function. Weibull was a contemporary of Palmgren and shared the results of his work with him. In 1947, Palmgren, in concert with G. Lundberg, also of Sweden, using strict series reliability analysis, incorporated his previous work along with that of Weibull, benchmarked to pre-1940 SKF rolling-element bearing tests, to form a probabilistic analysis to calculate rolling-element (ball and roller) bearing life (Lundberg and Palmgren (Refs. 11–12)). The Lundberg-Palmgren bearing life model is the basis for all contemporary bearing life calculations (Zaretsky (Ref. 6)).

Primary components limiting the life of gas turbine engines for aircraft application in the early 1950s were the ball and roller bearings used to support the main rotor shaft. At that time, the lives of these bearings were limited to approximately 300 h in aircraft turbine engine application. With improved bearing manufacturing and steel processing together with advanced lubrication technology, the potential improvements in bearing life can be as much as 80 times that attainable in the late 1950s or as much as 400 times that attainable in 1940 (Zaretsky (Ref. 6)).

B. L. Averbach and E. N. Bamberger (Ref. 13) examined approximately 200 incidents of bearings removed from aircraft engine service for cause. "The initial damage to these bearings was produced by abrasive particles, dents, grinding scores, skidding, large carbides and corrosion pits (p 241)." There was no classical subsurface-initiated spalling of any of the bearings reported. This would suggest that "classical rolling-element fatigue" is not a primary cause for bearing removal in aircraft turbine engine main rotor bearings. The issue becomes what the service lives of these bearings at a designated reliability are or the time at which these bearings are no longer fit for their intended application.

A review of aircraft bearing rejection criteria and causes was undertaken and reported in 1979 by J. S. Cunningham, Jr. and M. A. Morgan at the Naval Air Rework Facility, Cherry Point, North Carolina (Cunningham and Morgan (Ref. 14)). Their work is unique and, to the best of our knowledge, the only data of this type reported and available in the open literature. Their data were derived "from three 80-day engineering samples taken during 1969, 1971 and 1977 (p 435). Cunningham and Morgan (Ref. 14) concluded that rolling-element bearings "tend to fail at random intervals from corrosion, contamination, wear, or handling damage long before (rolling-element) fatigue initiates a spall (p 439)." From these data it is reasonable to conclude that the bearing service life is less than the calculated bearing life. Though no operating times are associated with the respective bearings associated with these data, it is possible to qualitatively associate a time related to each failure mode relative to the bearing calculated life.

In view of the aforementioned, the objectives of the work reported herein were to determine (a) the bearing service life as a function of the bearing L_{10} (fatigue) life; (b) bearing life as a function of each probable cause for removal; and (c) from commercial aircraft engine bearing field data, the percentage of rolling-element bearings removed for rolling element fatigue.

Statistical Method

Weibull distribution function. In 1939, W. Weibull (Refs. 9–10) developed a method and an equation for statistically evaluating the fracture strength of materials based upon small population sizes. This method has been applied to analyze, determine, and predict the cumulative statistical distribution of fatigue failure or any other phenomenon or physical characteristic that manifests a statistical distribution. The dispersion in life for a group of homogeneous test specimens can be expressed by:

$$\ln\ln\frac{1}{S}=m\ln\left(\frac{L-L_\mu}{L_\beta-L_\mu}\right)\text{ where } L_\mu<L<\infty;\ 0\le S\le 1 \tag{1}$$

Where, S is the probability of survival as a fraction ($0\le S\le 1$); m is the slope of the Weibull plot; L is the life cycle (stress cycles); L_μ is the location parameter or the time (cycles) below which no failure occurs; and L_β is the characteristic life (stress cycles). The characteristic life is that time at which 63.2% of a population will fail or 36.8% will survive (Zaetsky; Ref. 6)).

The format of Equation 1 is referred to as a three-parameter Weibull equation. For most — if not all — failure phenomena, there is a finite time period under operating conditions when no failure will occur. In other words, there is zero probability of failure, or a 100% probability of survival, for a period of time during which the probability density function is non-negative. This value is represented by the location parameter L_μ. Without a significantly large database, this value is difficult to determine with reasonable engineering or statistical certainty. As a result, L_μ is usually assumed to be zero and Equation 1 can be written as:

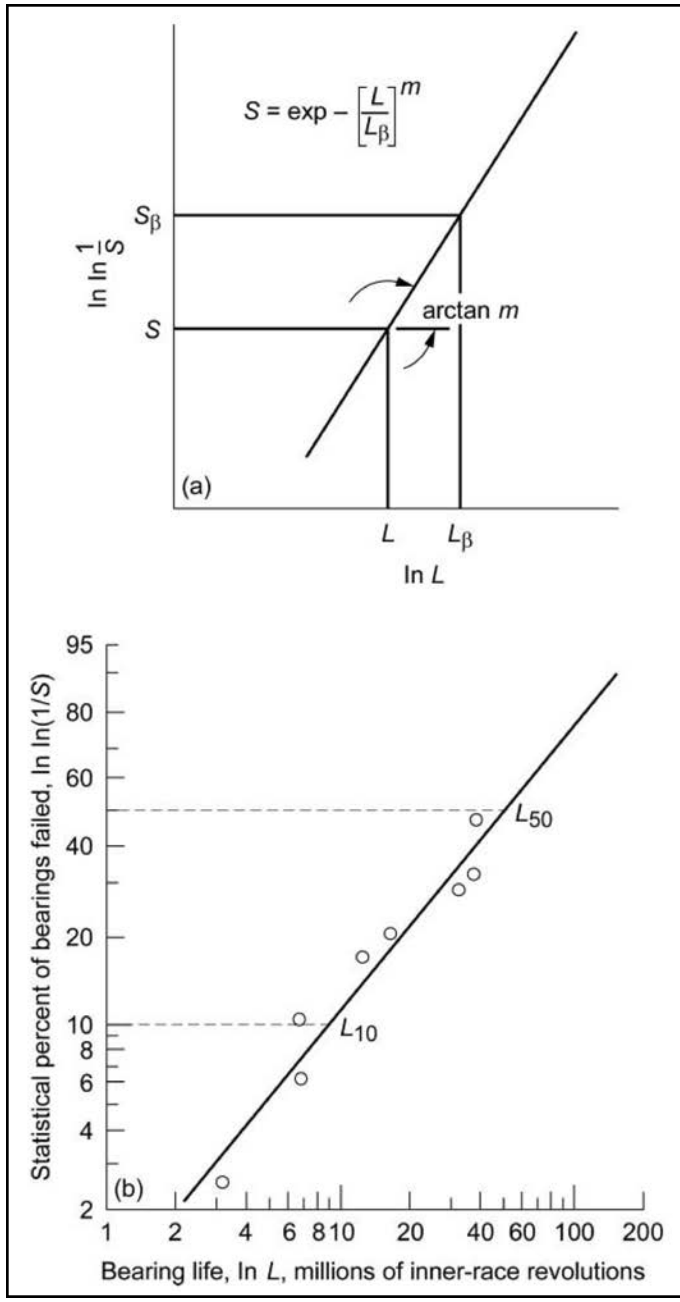

Figure 1 Weibull plot where (Weibull modulus) slope of tangent of line is m; probability of survival S is 36.8% at which $L=L_\beta$ or $L/L_\beta=1$. (a) Schematic where S is probability of survival. (b) Rolling-element bearing fatigue data where lnln(1/S) is presented in ordinate as statistical percentage of bearings failed (Zaretsky (Ref. 6)).

$$\ln\ln\frac{1}{S}=m\ln\left(\frac{L}{L_\beta}\right)\text{ where } 0<L<\infty;\ 0\le S\le 1 \tag{2}$$

This format is referred to as the two-parameter Weibull distribution function. The estimated values of the Weibull slope m and L_β for the two-parameter Weibull analysis will not in general be equal to those of the three-parameter analysis. As a result, for a given survivability value S, the corresponding value of life L will be similar but not necessarily the same in each analysis (Zaretsky; (Ref. 6)).

By plotting the ordinate scale as lnln $(1/S)$ and the abscissa

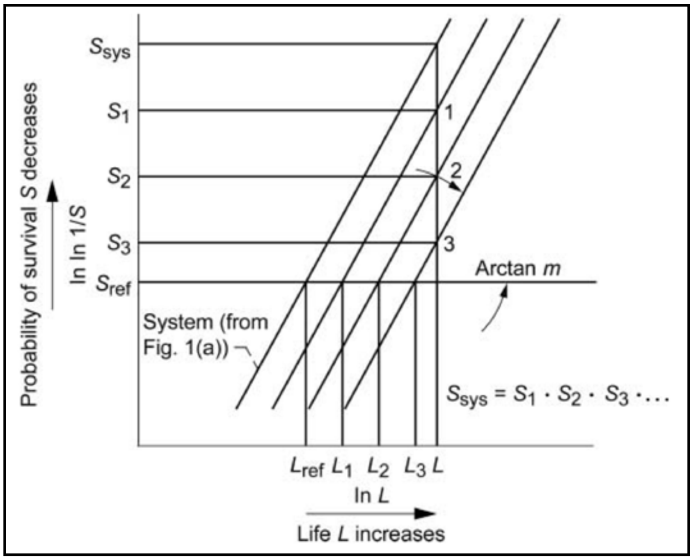

Figure 2 Sketch of multiple Weibull plots where each numbered plot represents the cumulative distribution of each component in the system, and the system Weibull plot represents the combined distribution of plots 1, 2, 3, etc. (all plots are assumed to have the same Weibull [modulus] slope *m*; (Zaretsky; Ref. 6).

scale as ln L, a two-parameter Weibull cumulative distribution will plot as a straight line, which is called a "Weibull plot." Usually, the ordinate is graduated in statistical percentage of specimens failed F where $F=[(1-S)\times 100]$. Figure 1a is a generic Weibull plot with some of the values of interest indicated. Figure 1b is a Weibull plot of actual bearing fatigue data (Zaretsky; (Ref. 6)).

L. G. Johnson (Ref. 15) developed methods for statistical data analysis utilizing the Weibull distribution function to define fatigue life population distribution.

Strict series reliability. If there are n independent components, each with a probability of the independent event (failure) not occurring equal to $(1–F)$, the probability of the event not occurring in the combined total of all components can be expressed as:

$$(1-F)^n = \exp - [nf(X)] \tag{3}$$

Equation 3 gives the appropriate mathematical expression for the principle of the weakest link in a chain or, more generally, for the size effect on failures in solids. The application of Equation 3 is illustrated by a chain consisting of several links. Testing finds the probability of failure F at any load X applied to a single link. To find the probability of failure F_n of a chain consisting of n links, one must assume that if one link has failed the whole chain fails. That is, if any single part of a component fails, the whole component has failed. Accordingly, the probability of non-failure of the chain $(1- F_n)$, is equal to the probability of the simultaneous non-failure of all the links. Thus,

$$1 - F_n = (1 - F)^n \tag{4a}$$

Or,

$$S_n = S^n \tag{4b}$$

Referring to Figure 2, where the probabilities of failure (or survival) of each link are not necessarily equal (i.e., $S_1 \neq S_2 \neq S_3 \neq \ldots$), for the probability of survival of the chain as a system, Equation 4b can be expressed as:

$$S_{sys} = S_1 \cdot S_2 \cdot S_3 \cdot \ldots \tag{4c}$$

Again referring to Figure 2, substituting appropriate values of the probability of survival S from Equation 2 into Equation 4c, where L_{ref} in Figure 2 is L_{serv} results in the following relation:

$$\left[\frac{1}{L_{serv}}\right] = \left\{\left[\frac{1}{L_1}\right]^m + \left[\frac{1}{L_2}\right]^m + \left[\frac{1}{L_3}\right]^m + \ldots\right\}^{1/m} \tag{5a}$$

$$1 = \left[\frac{L_{serv}}{L_1}\right]^m + \left[\frac{L_{serv}}{L_2}\right]^m + \left[\frac{L_{serv}}{L_3}\right]^m + \ldots \tag{5b}$$

Where,

$$X_1 = \left[\frac{L_{serv}}{L_1}\right]^m \tag{5c}$$

$$X_2 = \left[\frac{L_{serv}}{L_2}\right]^m$$

$$X_3 = \left[\frac{L_{serv}}{L_3}\right]^m$$

Table 1 Virtual rolling-element bearing fatigue database for generic angular-contact ball bearing subject to Weibull statistical analysis (Vleck, et al; Ref. 17).

No.	Time to failure (h)	Component failed	No.	Time to failure (h)	Component failed	No.	Time to failure (h)	Component failed
1	262	IR	21	2.933	IR	41	6.287	OR
2	476	IR	22	3.053	RE	42	6.564	IR
3	652	IR	23	3.181	IR	43	6.870	RE
4	803	RE	24	3.311	OR	44	7.211	IR
5	950	IR	25	3.444	RE	45	7.600	IR
6	1.090	OR	26	3.579	IR	46	8.053	OR
7	1.224	IR	27	3.717	RE	47	8.604	RE
8	1.354	IR	28	3.858	IR	48	9.316	RE
9	1.488	IR	29	4.003	OR	49	10.347	RE
10	1.600	OR	30	4.153	RE	50	12.408	OR
11	1.723	IR	31	4.306	IR			
12	1.845	RE	32	4.466	IR			
13	1.966	IR	33	4.630	RE			
14	2.086	RE	34	4.802	IR			
15	2.206	OR	35	4.981	OR			
16	2.321	RE	36	5.168	IR			
17	2.442	IR	37	5.368	IR			
18	2.563	RE	38	5.573	RE			
19	2.685	IR	39	5.795	IR			
20	2.809	OR	40	6.031	IR			

[a]IR = inner ring; number of inner-ring failures, 25. RE = rolling-element (ball); number of ball failures, 15. OR = outer ring; number of outer-ring failures, 10.

The fractional percentage X is related to each component that has failed in the system for a specific service life and reliability and assumes that the Weibull modulus, m, is the same for each component.

Unfortunately Equation 5 is only an approximation because the system Weibull modulus m can vary with sample size, operating conditions, and failure mode. In a balanced component life system, the system Weibull modulus, m, will be somewhere between the highest and the lowest of the components' Weibull slopes. A form of this equation can be solved numerically for system reliability as a function of life and plotted on Weibull coordinates (Savage, et al. (Ref. 16)). The resulting graph can be fitted with a best-fit straight line to determine the system Weibull slope and the system L_{10} life. In the event of an unbalanced life system, the lowest lived component will dominate the system failures and, thus, can serve as a good approximation for the system Weibull properties. However, at a given reliability the system life will always be lower than the lowest lived component because other components can also fail.

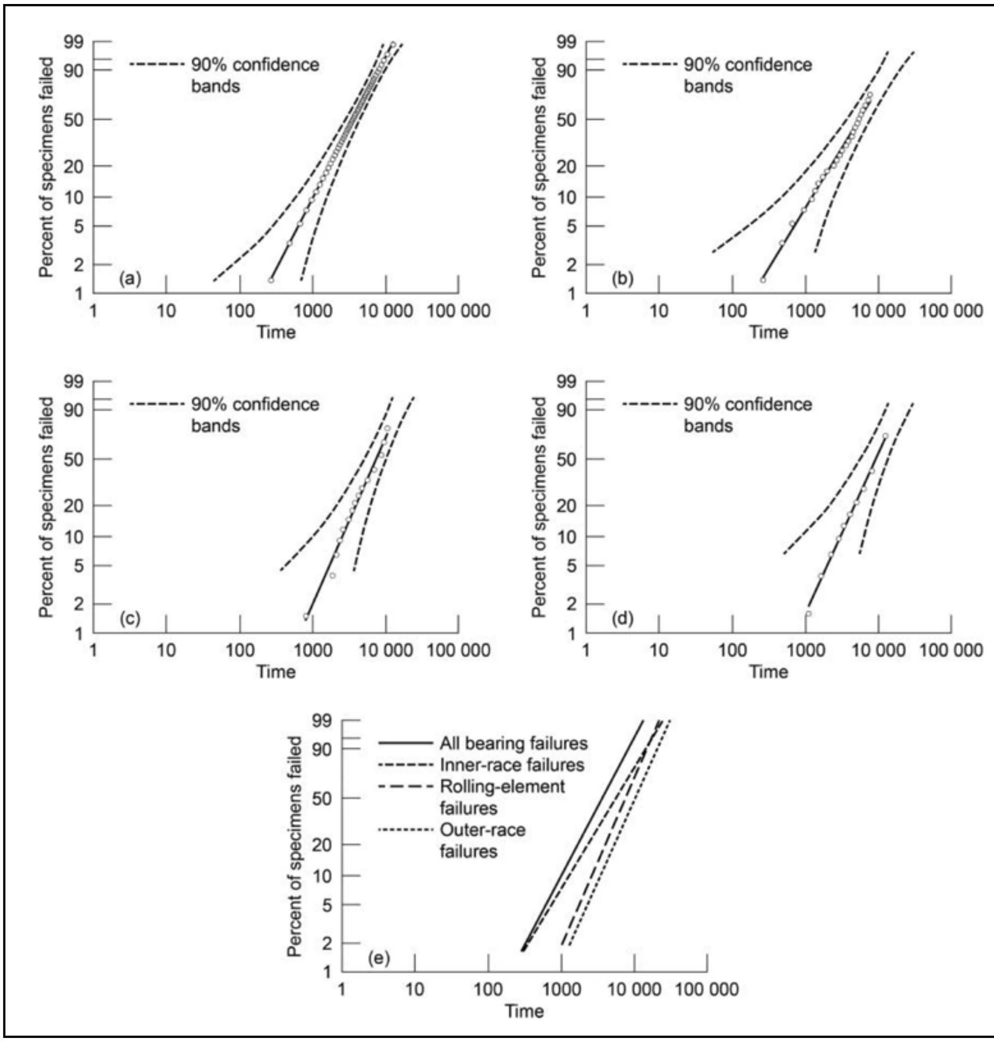

Figure 3 Failure distribution of generic angular-contact ball bearings, virtually tested under pure thrust band. (a) All bearing component failures, failure index 50 out of 50. (b) Inner race failures, failure index 25 out of 50. (c) Rolling-element failures, failure index 15 out of 50. (d) Outer race failures, failure index 10 out of 50. (e) Summary.

Application of strict series reliability to bearing fatigue. Assume, based on the work of Vlcek et al. (Ref. 17), that a population of 50 generic angular-contact ball bearings is virtually tested under pure thrust load. It is further assumed that the failure mode for these bearings is classical subsurface rolling-element fatigue. Their failure times and the respective component, inner ring (IR), ball (B), or outer ring (OR), that failed in each bearing are summarized in Table 1. For the purpose of this example, the failure of each component in the bearing is considered the failure time of the entire bearing; these data were analyzed using the method of L. G. Johnson (Ref. 15). The 90% confidence bands are shown with respect to these data. This would mean that in 90% of all possible cases, it can be expected, with reasonable statistical certainty, that the failure data points and thus the failure population distribution will fall between these confidence bands. The results are shown in the Weibull plot of Figure 3a and are summarized in Table 2.

In order to determine the lives of each of these respective components in the system, the failure times for a specific component being analyzed are considered a failure, and the failure times for the other components are considered to be non-failures or suspensions. These components are consid-

Table 2 Summary of life analysis for virtual rolling-element fatigue data for generic angular-contact ball bearing.

	Life (h)		Weibull modulus,
	L_{10}	L_{50}	m
Weibull analysis (data from Fig. 3)			
Total bearings	999′	3,526	1.49
Inner ring	1,226	5,418	1.27
Rolling-elements	2,517	7,305	1.77
Outer ring	2,981	9,077	1.69
Strict series reliability (analysis benchmarked to Fig. 3a)			
Total bearings	999*	3,526	1.49
Inner ring	1,591	5,633	1.49
Rolling-elements	2,241	7,935	1.49
Outer ring	2,942	10,416	1.49
Strict series reliability (analysis benchmarked to Fig. 3d)			
Total bearings	1,150	3,503	1.69
Inner ring	1,733	5,279	1.69
Rolling-elements	2,345	7,143	1.69
Outer ring	2,981*	9,077	1.69

*Analysis benchmarked to component L_{10} life and Weibull modulus m.

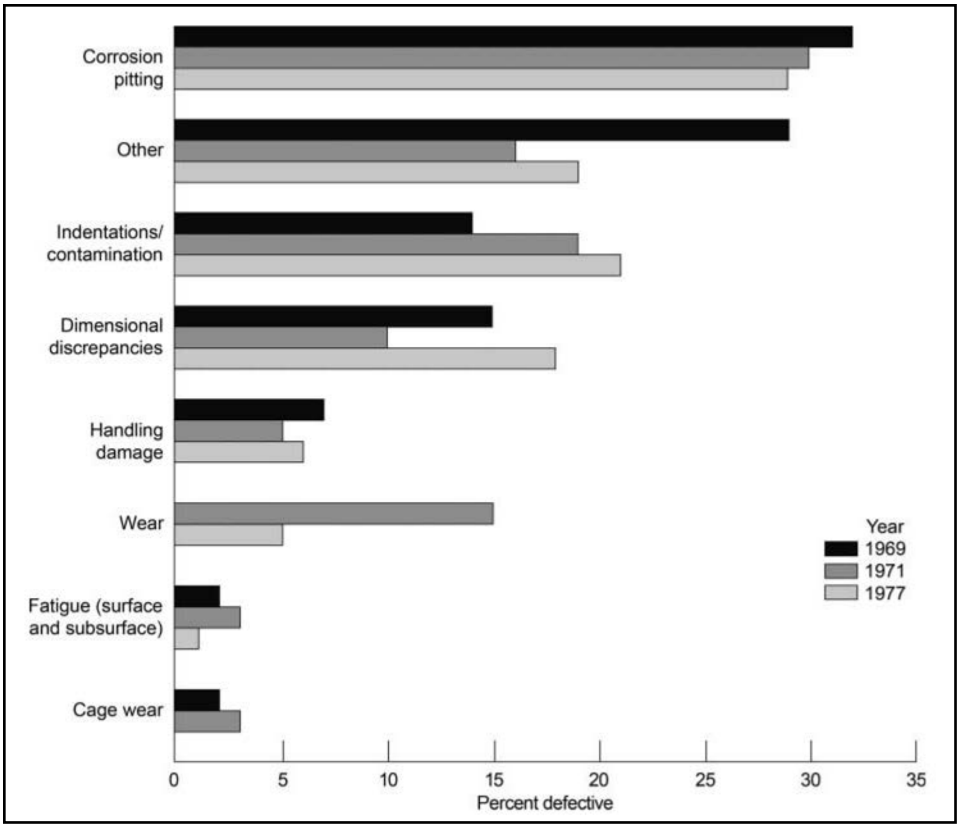

Figure 4 Naval Air Rework facility rolling-element bearing data for bearings removed from service for cause for three 80-day periods during 1969, 1971, and 1977 (data from Cunningham and Morgan; Ref. 14).

ered suspensions because the bearing would have continued to operate for an unknown time if they had not been removed from test when they failed. Again, using the method of Johnson (Ref. 15), the Weibull plots for the inner ring, balls, and outer ring are shown in Figures 3b to 3d, respectively. The L_{10} and L_{50} lives and the Weibull modulus m are summarized in Table 2 under the column designated "Weibull analysis." The life and reliability of the system cannot exceed the life and reliability of the lowest lived component in the system — whether it is the inner ring, ball, or outer ring.

For purpose of example, assume that the data of Table 1 were available without designating the failed component in each bearing. However, the percentage of the failures representing the inner ring, balls, and outer ring is known. Using strict series reliability from Equation 5c and the data from Figure 3a, the L_{10} lives of the inner ring, balls, and outer ring are calculated. The L_{50} lives are calculated using Equation 2; the L_{10} and L_{50} lives and the Weibull modulus m are summarized in Table 2 under the "Strict series reliability" benchmarked to the total bearing L_{10} life and Weibull modulus of 1.49. These values fall within the 90% confidence bands of Figure 3.

Another example: if it is assumed that the only data that are available are those shown in Figure 3d for the outer race and the percentage of the failed population that it represents, it is possible to use strict series reliability to calculate the lives of the entire bearing using Equation 5c. The L_{50} lives are calculated using Equation 2. The L_{10} and L_{50} lives and the Weibull modulus m are summarized in Table 2 under the "Strict series reliability" benchmarked to the total bearing L_{10} life and Weibull modulus of 1.69. These values fall within the 90% confidence bands of Figure 3.

We define *bearing failure* as the time at which the bearing is no longer fit for its intended purpose — even though the bearing is still functioning. This would be considered a cause for removal. In the above examples, if it is assumed that each of the components that failed represents a different failure mode instead of the specific component, it is possible to use Weibull statistical analysis and/or strict series reliability to determine the service life of the entire bearing set and/or the resulting life at a given reliability (probability of failure) for each failure mode represented with reasonable engineering and statistical certainty.

Results and Discussion

Naval Air Rework Facility rolling-element bearing data. J. S. Cunningham, Jr. and M. A. Morgan of the Naval Air Rework Facility, Cherry Point, North Carolina (Cunningham and Morgan (Ref. 14)) published data for rolling-element bearings removed from service for cause for three 80-day periods during 1969, 1971, and 1977. These data were presented by Cunningham and Morgan at the 33rd meeting of the ASLE (now STLE) in Dearborn, Michigan, April 17–20, 1978, and published a year later in Cunningham and Morgan (Ref. 14).

In the Introduction to their paper, Cunningham and Morgan (Ref. 14) state:

> Extensive time and effort has been devoted to calculation of (rolling-element) bearing (L_{10}) life, to determination of cage instability and to studies of the effects of various lubricants and protective coatings. However, the researcher is often at a loss for documented data on bearing rejections in a "real world" environment. This information is essential to determine those areas of developmental work that will produce the most significant increases in actual bearing (service) life and reliability. A bearing with a design life of 5,000 hours is of little value if its operational environment contributes to excessive corrosion pitting at 500 hours (p 435)."

The data of Cunningham and Morgan (Ref. 14) are summarized (Fig. 4). They categorize the probable causes of failure as 1) fatigue (surface and subsurface origin); 2) cage wear; 3) wear; 4) handling damage; 5) dimensional discrepancies; 6) debris denting and contamination; 7) corrosion pitting; and 8) other (common failure modes). From Zaretsky (Ref. 18), the other common failure modes include 1) misalignment; 2) true and false brinelling; 3) excessive thrust; 4) heat and

Table 3 Assumed probable causes for bearing removal, their related percentage as a function of bearings that have failed, and related L_{10} life based on a bearing service life at a 90% reliability where L_{serv} = 206 h. Assumed bearing L_{10} (fatigue) life = 5,000 h*

Cause for removal	Percentage of bearings failed related to cause for removal	Calculated L_{10} life (h)
Fatigue (surface and subsurface origin)	3	5,000
Cage wear	3	5,000
Wear	6	2,659
Handling damage	7	2,311
Dimensional discrepancies	17	1,031
Debris denting and contamination	20	890
Corrosion pitting	27	677
Other	17	1,031

* Weibull modulus *m* was assumes equal to 1.1 for all causes of removal.

thermal preload; 5) roller edge stress; 6) cage fracture; 7) element or ring fracture; 8) skidding; and 9) electric arc discharge. In all, there are 16 probable causes for bearing failure and/or removal wherein the bearing is no longer fit for its intended purpose but can still be operational. Good engineering and maintenance practice would suggest that these bearings be removed from service when the determination is made that they are no longer fit for their intended purpose. It is probable that if these data were taken today, the categories outlined above and/or their related percentages would be different. Unfortunately, individual rolling-element bearing types and related times to removal are not provided for these data. Other data of this type, if it exists, are not provided in the open literature.

Cunningham and Morgan (14) observe that:

> Bearing failures due to spalling are rare and almost insignificant to the overall rejection rate. Furthermore, examination of the overall rejection rate under this category revealed corrosion to be a possible cause of spall origin. Classical fatigue seems to play a very minor role in bearing reliability problems. In most cases, bearing failures are random and do not display a defined time relationship. As a result, many non-safety components are allowed to continue in service as long as they function properly (p 437).

However, using the Cunningham and Morgan (Ref. 14) database, it is possible — using Weibull statistical analysis and strict series reliability — to determine the bearing service life as a function of the bearing L_{10} (fatigue) life and bearing life as a function of each probable cause for removal. It should be noted that rolling-element fatigue, whether of surface or subsurface origin, accounts for 3% or less of the bearings removed from service for cause. That is, they were unfit for their intended purpose at the time of removal.

In order to determine and/or assign a qualitative life and resultant life factor from Figure 4, Table 3 lists probable causes for removal given to a hypothetical bearing having a design (L_{10}) life of 5,000 h, as per the example above from Cunningham and Morgan (Ref. 14).

From Equation 5c for fatigue as the failure origin where $X = 0.03$, $L_{10} = 5{,}000$ h, and $m = 1.1$,

$$X = \left[\frac{L_{serv}}{L_{10}}\right]^m = \left[\frac{L_{serv}}{5000}\right]^{1.1} = 0.03 \tag{6a}$$

$$L_{serv} = 206 \text{ h.} \tag{6b}$$

If we apply Equation 5c for corrosion as a cause for removal where $X = 0.27$, $L_{serv} = 206$ h, and $m = 1.1$,

$$X_1 = \left[\frac{L_{serv}}{L_{10}}\right]^m = \left[\frac{206}{L_{10,c}}\right]^{1.1} = 0.27 \tag{7a}$$

$$L_{10,c} = 677 \text{ h.} \tag{7b}$$

For purposes of discussion, if we had selected a Weibull modulus $m = 1.5$ in Equation 7a, the resultant bearing life, $L_{10,c}$, based on corrosion would be 493 h.

Using a bearing service life $L_{serv} = 206$ h from Equation 6 and a Weibull modulus $m = 1.1$, the L_{10} lives were calculated for each cause for removal. These values are given in Table 3 and the respective Weibull plots are shown (Fig. 5). As previously discussed, this analysis is benchmarked to the assumed bearing L_{10} fatigue life of 5,000 h.

Figure 6 shows the percentage of bearings removed from service for cause based on the calculated service life but benchmarked to the bearing L_{10} fatigue life of 5,000 h. This analysis shows that the percentage of bearings in service

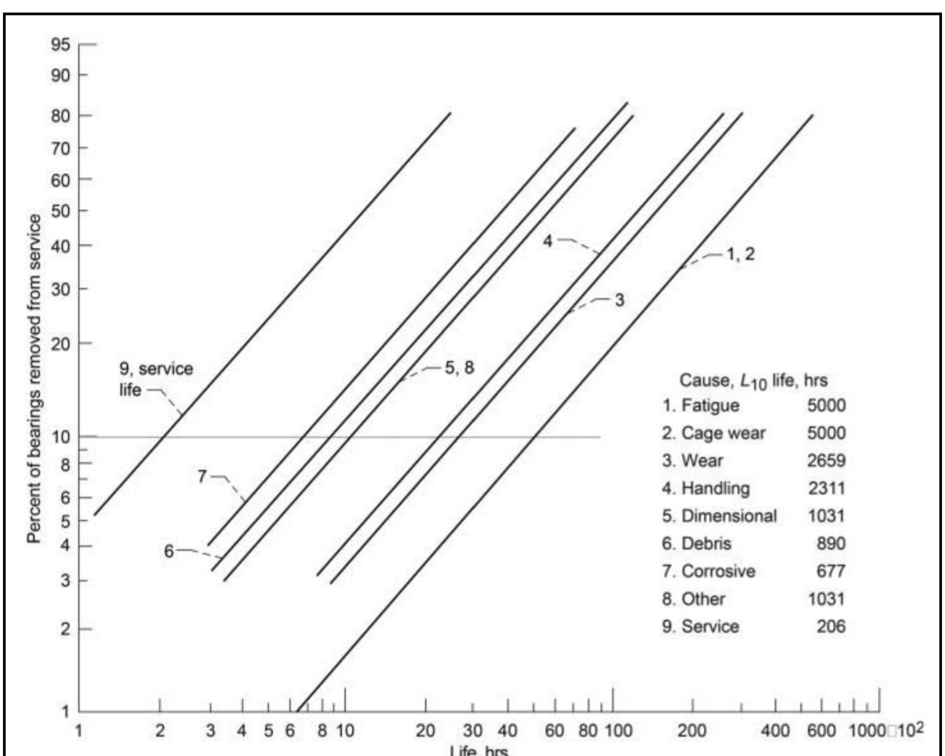

Figure 5 Rolling bearing service life and life distribution based upon cause for removal where the calculated L_{10} bearing life is based on rolling-element fatigue equal to 5,000 h.

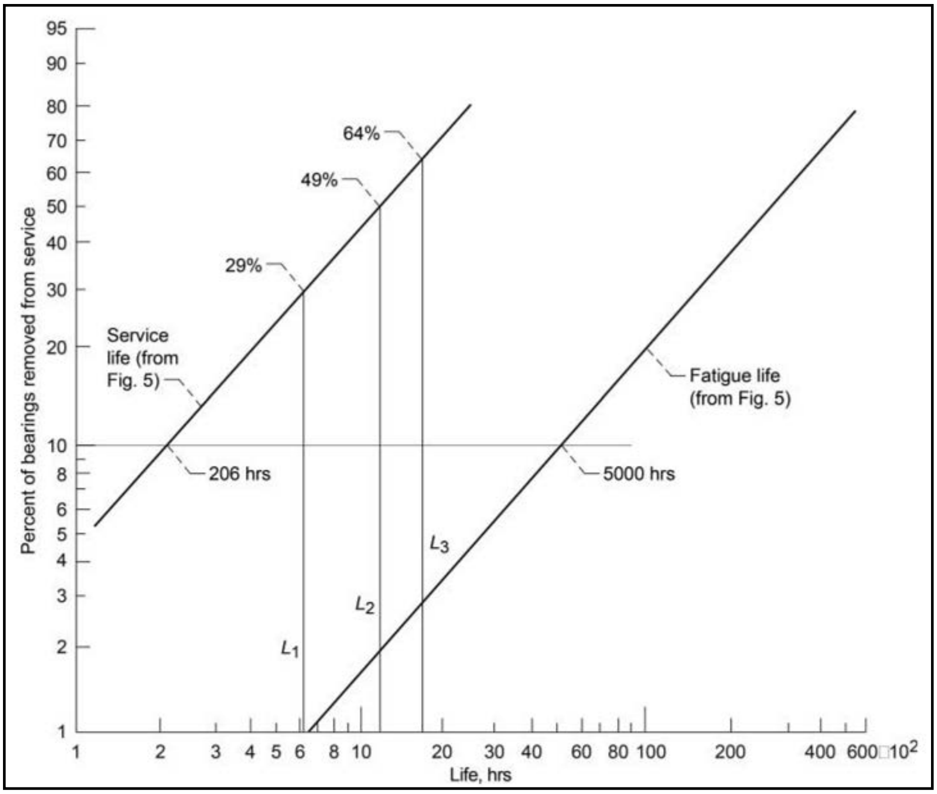

Figure 6 **Percentage of bearings removed from service for cause as being unfit for their intended purpose, based on the calculated service life but benchmarked to the bearing L_{10} fatigue life of 5,000 h.**

would be removed as being unfit for their intended purpose as follows:

1. At approximately 591 h, or the bearing L_1 fatigue life (12% of the L_{10} fatigue life), 29% of the bearings would be removed from service.
2. At approximately 1,114 h, or the bearing L_2 fatigue life or (22% of the L_{10} fatigue life), 49% of bearings would be removed from service.
3. At approximately 1,618 h, or the bearing L_3 fatigue life or (32% of the L_{10} fatigue life), 64% of bearings would be removed from service.
4. At 5,000 h, or the bearing L_{10} fatigue life, 97% of the bearings would be removed from service.

The above analysis would suggest that the anecdotal perception that most bearings are removed from service before reaching their L_{10} fatigue or catalog life has merit.

An issue remains regarding this analysis. What would the service life of the bearing be if fatigue (both surface and subsurface) were to be eliminated as a failure mode? Using Equation 5a and the L_{10} lives for each mode of failure from Table 3, and eliminating fatigue as a failure mode for this calculation, the bearing service life L_{serv} increases from 206 to 212 h. This would suggest that by eliminating rolling-element fatigue as a cause for removal, the service life of these bearings would be increased by 3%.

In Table 3 we assume the Weibull modulus $m = 1.1$ and is a constant for all failure modes. As we previously discussed under Strict Series Reliability, Equation 5a is only an approximation because the system Weibull modulus m is a variable based on failure mode and is not necessarily a constant, as assumed for the above analysis. In a balanced component life system, the service life modulus m is somewhere between the highest and the lowest of the Weibull modulus m for each of the failure modes. Hence, if we knew the Weibull modulus m for each failure mode, the life analysis could be solved numerically for system reliability as a function of life, and plotted on Weibull coordinates (Savage, et al. (Ref. 16)). The resulting graph can be fitted with a best-fit straight line to determine the system Weibull slope and the service life at a 90% reliability or a service L_{10} life.

In the event of an unbalanced life system, the lowest lived failure mode will generally dominate bearing failures and, thus, can serve as a good approximation for the system Weibull properties. From Table 3, "Corrosion pitting" is the lowest-lived failure mode. However, at a given reliability the service life will always be lower than that caused by the lowest-lived failure mode because other failure modes can also result in bearing removal.

Commercial aircraft turbine engine bearings. As previously discussed, Averbach and Bamberger (Ref. 13) examined approximately 200 incidents of bearings removed from aircraft engine service for cause. "The initial damage to these bearings was produced by abrasive particles, dents, grinding scores, skidding, large carbides and corrosion pits" (p 241). There was no classical subsurface- or surface-initiated spalling of any of the bearings reported. As with the work of Cunningham and Morgan (Ref. 14), this would suggest that classical rolling-element fatigue is not a primary cause for bearing removal in aircraft turbine engine main rotor bearings. The issue becomes what the service lives of these bearings are at a designated reliability or the time at which these bearings are no longer fit for their intended application.

For several decades it has been the practice of most, if not all, airlines to rework their engine rolling-element bearings when their engines are sent for refurbishment or rework (overhaul) and the bearings are removed from service. In general, most first-run commercial aircraft engines are removed from service between 15,000 and 20,000 h of operation. The rolling-element bearings are removed from the engine and are subjected to Level I or Level II rework (Zaretsky and Branzai (Ref. 19)). According to Zaretsky and Branzai (Ref. 19), Level I repair is a reclamation of the bearings that involves inspecting a used bearing and checking and comparing it with new bearing data or reverse-engineering data requirements. Other Level I processes include, but are not limited to, demagnetization, cleaning, nondestructive testing, visual/microscopic inspection, and minor repairs. At Level I inspections the bearing can be rejected for cause as being no longer fit for its intended purpose. For each Level I repair the resulting bearing life is reduced from that of a new or unused bearing.

For those bearings that require repair beyond that of the Level I and are discarded for cause, the Level II repair is used, which encompasses all of the operations of Level I plus one or more of the following (Zaretsky and Branzai (Ref. 19)):

1. Replacing rolling-elements (with new ones)
2. Rework or replacing retainers (cages)
3. Interchanging used components and/or substituting new components to create a different assembly identity
4. Grinding or polishing and/or plating mounting surfaces as necessary to return to original drawing dimensions
5. Honing (superfinishing) raceways (to the maximum oversized rolling-element allowed)

Zaretsky and Branzai (Ref. 20) established a simple algebraic relationship to determine the L_{10} rolling bearing fatigue life of bearings subject to rework. Depending on the extent of the repair, and based on theoretical analysis, representative life factors (LFs) for bearings subject to repair that ranged from 0.87 to 0.99 the lives of new bearings. According to Zaretsky and Branzai (Ref. 20), the potential cost savings from bearing rework varies from 53 to 82% that of new bearings, depending on the cost, size, and complexity of the bearing.

Timken Aerospace Bearing Repair, Los Alamitos, California (formerly Bearing Inspection, Inc.) furnished us with their rolling-element bearing repair (rework) history for the period January 2007 through December 2013. These data included approximately 224,000 aircraft engine ball and roller bearings repaired that included the data for two aircraft engine types designated by us as Engine Type Series A and Engine Type Series B. In general these bearings are manufactured from vacuum arc re-melted and/or vacuum induction melted–vacuum arc re-melted AISI 52100 and AISI M-50 bearing steels. In addition, these bearings operate, for the most part, under a lubricant film parameter 1.5 with lubricant (oil) filtration $\beta_{x(C)} \leq 10$. These data are summarized in Table 4; unfortunately, it is not categorized by bearing type and size, engine main shaft position, or cause for rejection, except for fatigue. Of the 224,000 bearings reported in Table 4A, 1,977 bearings or ~0.88% (<1%) were rejected for fatigue. The specific bearing component of these 1,977 that failed from fatigue is identified in Table 4B. Unfortunately, the percentage or number of bearings removed from service for reasons other than fatigue were not available.

Though we do not have information and/or data that would allow us to segregate the bearings by type, application, and/or time, it can be reasonably assumed that the main shafts of the two aircraft engines represented in Table 4, from which the bearings were removed, had a set of seven rolling-element bearings each — two each angular-contact ball bearings and five each cylindrical roller bearings. From strict series reliability (Eqs. 4 and 5), the bearing system life calculated will be less than the lowest lived bearing in the assembly. This is assumed to be the engine main shaft angular-contact ball

Table 4 Commercial aircraft engine rolling-element bearing rework history, from January 2007 thru December 2007*

A. Number of bearings rejected for fatigue for all engine bearings				
Engine type All series	Total number of bearings received~224,000	Total number of bearings rejected Unknown	Number of bearings rejected for fatigue 1,977 out of ~224,000	Fatigue rejection ratio (%) 0.88
B. Bearings rejected for fatigue per bearing component for all engine bearings				
Engine type	Bearing component	Number of bearings rejected for fatigue	Fatigue rejection	
All series	Undesignated	107	0.05	
	Rolling-elements	533	0.24	
	Inner ring	791	0.35	
	Outer ring	546	0.24	
	Total	1,977 out of 224,000	0.88	
C. Bearings removed from engine designation A				
Engine type	Total number of bearings received	Number of bearings rejected for all reasons	Rejection ratio for all reasons (%)	
Series A	24,471 out of ~224,000	5,049	20.6	
D. Bearings removed from engine designation A				
Engine type	Total number of bearings received	Number of bearings rejected for all reasons	Rejection ratio for all reasons (%)	
Series AI	9,184 out of 24,471	1,613	17.6	
E. Bearings rejected for fatigue per bearing component from engine designation A				
Engine type	Bearing component	Number of bearings rejected for fatigue	Fatigue rejection ratio (%)	
Series AI	Undesignated	0	0	
	Rolling-elements	0	0	
	Inner ring	3	~0.04	
	Outer ring	14	~0.15	
	Total	17 out of 9,184	~0.19	
F. Bearings removed from engine designation B				
Engine type	Total number of bearings received	Number of bearings rejected for all reasons	Rejection ratio for all reasons (%)	
Series B	1,525 out of 224,000	252	16.5	
G. Bearings rejected for fatigue per bearing component from engine designation B				
Engine type	Bearing component	Number of bearings rejected for fatigue	Fatigue rejection ratio (%)	
Series B	Undesignated	0	0	
	Rolling-elements	0	0	
	Inner ring	6	~0.04	
	Outer ring	1	~0.01	
	Total	7 out of 1,525	~0.05	

*Courtesy of Timken Aerospace Bearing Repair, Los Alamitos, California.

thrust bearing. It is further assumed that all bearings were removed from service on or before 20,000 engine operating hours.

Referring to Table 4C, of the 224,000 bearings reported in Table 4A, there were a total of 24,471 bearings removed for rework and inspected from what we designate as Engine Type Series A. Of this number 5,049 — or 20.6% — were rejected for cause. The data does not report the number of bearings comprising the 5,049 that had failed from fatigue. However, a sub-set of these data comprising 9,184 of the 24,471 bearings that we have designated as Engine Type Series A bearings are summarized in Tables 4D and 4E. From this group, out of the 1,613 bearings rejected for all causes, 17 individual bearings — or ~0.19% — were removed for fatigue.

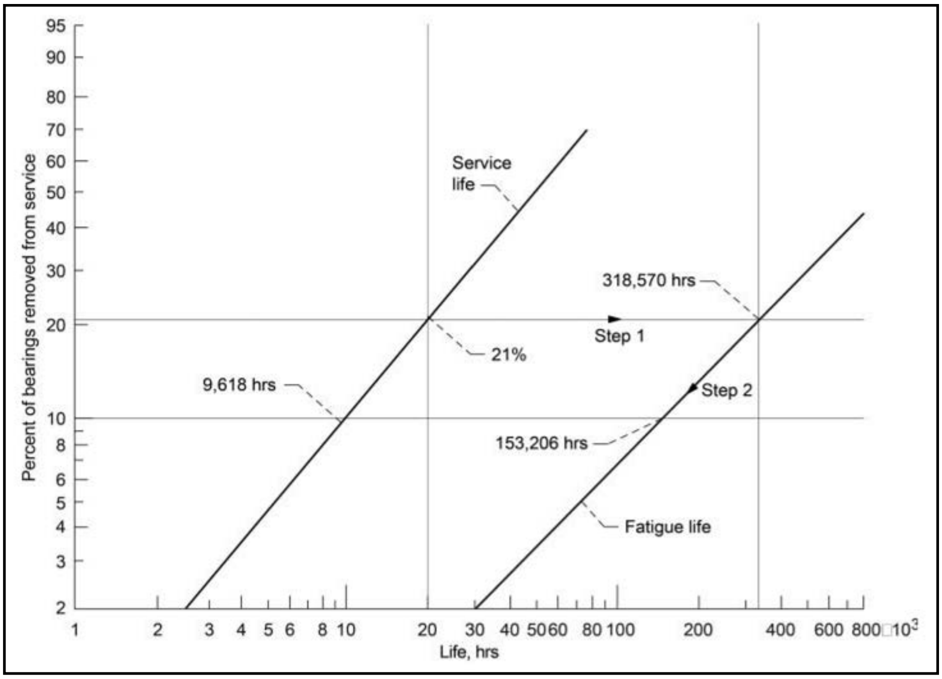

Figure 7 Aircraft engine bearing service and fatigue lives based on probable cause for removal and Weibull modulus, *m*, of 1.1 for 224,000 commercial aircraft engine ball and roller bearings over a period of 7 years.

Tables 4F and 4G contain bearing data for a different engine that we designate as Engine Type Series B. This data set includes 1,525 bearings, of which 252 — or ~16.5% — were rejected for cause. Of the 252 bearings rejected for cause, 7 — or ~2.8% of the bearings removed for cause — were rejected for fatigue.

Based upon the above discussion, for purposes of analysis it was assumed that all bearings were removed from service on or before 20,000 engine operating hours. Further, based on Table 4C it can be assumed that ~21% of all bearings were removed from service for cause. In addition, based on Table 4A, 1% of all bearings removed for rework failed from fatigue. This would imply for purposes of analysis that of all of the bearings that were removed for cause — approximately 5% {[0.01(224,000) ÷ 0.21(224,000)] × 100 = 4.76%} were for rolling-element fatigue.

Referring to the Weibull plot in Figure 7, a 21% service life (L_{21}) is shown. For fatigue failures it can be reasonably assumed for purposes of calculation that the Weibull modulus (slope) is equal to 1.1. Using Equation 5c and a Weibull modulus of 1.1, the L_{21} fatigue life is calculated to be 318,570 h (Step 1). From the Weibull distribution function — Equation 2 — the bearings' L_{10} fatigue lives equal 153,206 h (Step 2).

Again, referring to the Weibull plot (Fig. 7), a 21% service life (L_{21}) is shown together with an assumed Weibull modulus $m = 1.1$. We do not have data to determine the distribution (Weibull modulus m) for the population of bearings removed from service for all causes. However, we can reasonably assume, for purposes of engineering analysis, that the statistical distribution of the bearings that are removed from service for all causes can vary between the exponential distribution (Weibull modulus $m = 1$), the Raleigh distribution (Weibull modulus $m = 2$), and the normal or Gaussian distribution (Weibull modulus $m = 3.57$). From the Weibull distribution function (Eq. 2), the calculated L_{10} service lives equal 9,618, 13,517, and 15,985 or 6.3, 8.7, and 10.4% of the L_{10} fatigue lives, respectively. These results are shown in Figure 8. It can reasonably be concluded that most conservative bearing L_{10} service life calculation is obtained assuming an exponential distribution where we assumed $m = 1.1$ (Fig. 7).

From Equation 6 the bearing L_{10} service life can benchmarked and calculated to the bearing L_{10} fatigue life as follows:

$$X = \left[\frac{L_{serv}}{L_{10}}\right]^m \tag{8a}$$

$$L_{serv} = X^{1/m} L_{10}, \tag{8b}$$

Where, in Equations 8a and 8b, L_{serv} is the service life at a 90% reliability or a 10% probability for bearing removal; X is the number of bearings that were removed from service because of fatigue divided by the total of all bearings removed from service regardless of cause; and L_{10} is the bearing calculated life based on fatigue at a 90% reliability or a 10% probability of fatigue failure.

An issue that is unanswered from the above analysis is the suggested correlation between the bearing location parameter, L_μ, based on rolling-element fatigue and the L_{10} bearing service life using a Weibull modulus of 1.1. From Equation 1 and the work of Tallian (Zaretsky; Ref. 18) and Tallian (Ref. 21), it can be reasonably assumed that the location parameter L_μ, or the time below which no bearing fatigue failure should occur, is 0.053 L_{10} or for the commercial engine data, (0.053 × 153,206 h =) 8,120 h. From Equation 8b:

$$\text{Let } X^{1/m} = 0.053 \tag{8c}$$

$$\begin{gathered} m = 1.1 \\ X \approx 0.04 \end{gathered} \tag{8d}$$

At a 90% reliability, where 10% of all the bearings in service are removed from service for cause, 4% of those bearings that were removed are because of fatigue or 0.4% of all the bearings in service at that point in time. This compares to the 3% for the Naval Air Rework Facility bearing data and the ~5% commercial aircraft engine bearing rework data. Such a correlation at this time is speculative; more data are required. However, if such a correlation were to exist, it would greatly simplify the rolling-element bearing service life calculation.

General Comments

In the early years of the 20th century, rolling-element fatigue was the major cause for rolling-element bearing removal and limited the life and reliability of these bearings. Sadeghi et al. (Ref. 22) provide an excellent review of this failure mode.

Beginning with John Goodman (Ref. 5) and Arvid Palmgren (Ref. 2), the bearing industry has based the selection and sizing of these bearings on this failure mode. In the early gas turbine engines, engine life and reliability were linked to the fatigue life of those rolling-element bearings incorporated in the engine. Anecdotally, the life of these early engines and, thus, their bearings were limited to approximately 300 h. This can be compared to the estimated bearing fatigue life of over 100,000 h for the commercial aircraft engine bearings reported herein. Hence, the pre-1960 bearing service life was in fact the calculated bearing L_{10} fatigue life.

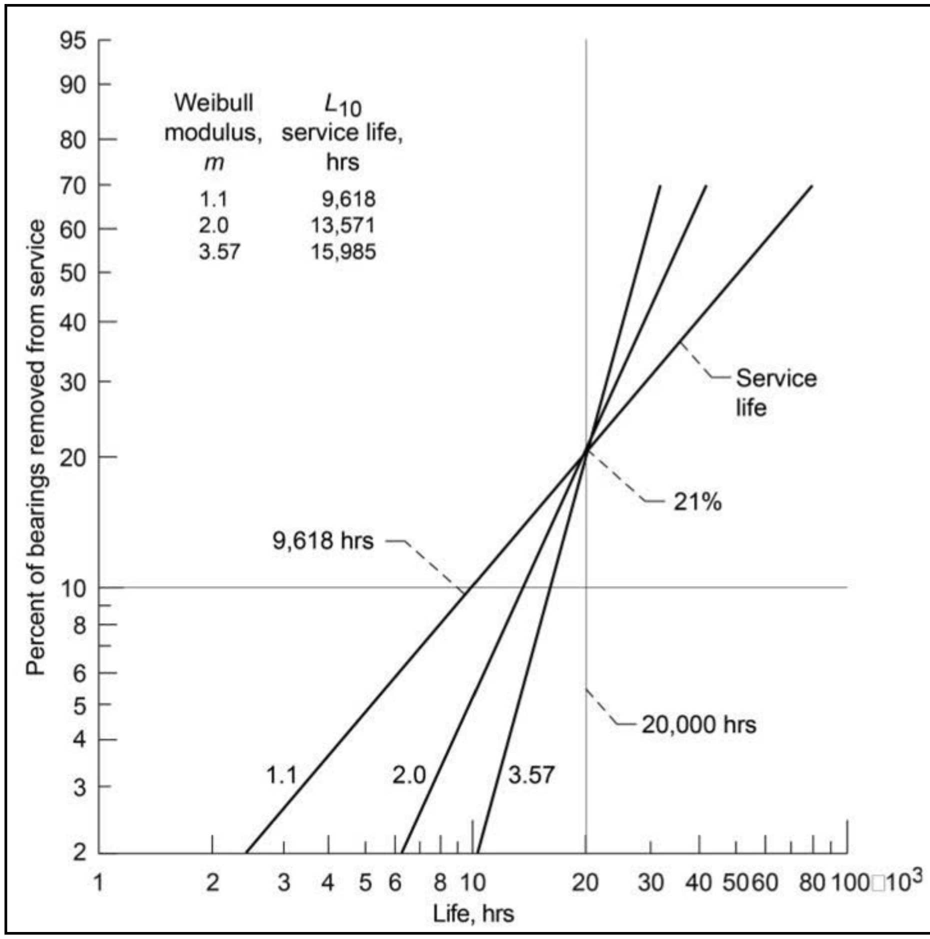

Figure 8 **Effect of Weibull modulus, *m* (statistical distribution), on engine bearing service life.**

In the early years of the bearing industry, acid and base refractory air-melting methods were used to process steel. Major advances in steel processing have occurred, beginning in the 1950s with the introduction of vacuum-melting procedures that significantly increased the bearing fatigue life (Zaretsky; Refs. 18 and 23).

By the early 1960s bearing fatigue life increased approximately five times that upon which Lundberg and Palmgren (Refs. 11–12) benchmarked their life model to (Zaretsky: Ref. 18). By 1992 the bearing fatigue life was approximately 200 times that benchmarked by Lundberg and Palmgren; and with improved manufacturing techniques, heat treatment procedures, and lubricants, the bearing fatigue life can be as much as 400 times the Lundberg-Palmgren calculation.

Though bearing fatigue life has significantly improved, the other failure modes and/or causes for removal have remained relatively speaking unchanged and application dependent. The bearing removal and replacement rate may not be significantly better than that in the early 1960s. It is suggested that bearing removal rate is application-dependent. There is no analytical method for individually calculating the respective replacement rates and/or life except by accumulating a database from field experience. Though a bearing may no longer be fit for its intended purpose for reasons other than fatigue, it may operate for extended periods of time in an application without causing secondary damage. However, once the application is shut down, reasonably prudent engineering and maintenance procedures would suggest that the bearing(s) be removed from service and replaced.

Summary of Results

In 1947 and 1952, G. Lundberg and A. Palmgren developed what is now referred to as the Lundberg-Palmgren model for rolling bearing life prediction based on classical rolling-element fatigue. Today, bearing fatigue probably accounts for less than 5% of bearings removed from service for cause. A bearing service life prediction methodology and tutorial indexed to eight probable causes for bearing removal, including fatigue, are presented, which incorporate strict series reliability; Weibull statistical analysis; available published field data from the Naval Air Rework Facility; and ~224,000 rolling-element bearings removed for rework from commercial aircraft engines. The following results were obtained:

1. Bearing service life L_{serv} can be benchmarked and calculated to the bearing L_{10} fatigue life as follows:

$$L_{serv} = X^{1/m} L_{10}$$

   where, L_{serv} is the service life at a 90% reliability or a 10% probability for bearing removal; X is a fractional percentage calculated by taking the number of bearings removed from service because of fatigue, divided by the number of all bearings removed from service, regardless of cause; m is the Weibull modulus of all of the bearings removed from service; and L_{10} is the bearing calculated life based on rolling-element fatigue at a 90% reliability — or a 10% probability of a fatigue failure.

2. The most conservative bearing L_{10} service life calculation is obtained assuming an exponential distribution where $m = 1.1$.
3. Of the ~224,000 commercial engine bearings removed from service for rework, 1,977, or 0.88%, were rejected because of fatigue.
4. From the Naval Air Rework Facility bearing data, eliminating rolling-element fatigue as a cause for removal, the L_{10} service life of these bearings would increase by approximately 3%. At 5,000 h or the bearing L_{10} fatigue life, 97% of the bearings would be removed from service for cause. **PTE**

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