Influence of Gear Loads on Spline Couplings

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Involute splines are commonly used in gearboxes to connect gears and shafts, especially when high torque is transmitted through the coupling. The load is shared among multiple teeth around the coupling circumference, resulting in higher load capacity than a conventional single key. However, the total load is not equally shared among all spline teeth, mainly because of pitch deviations resulting from the manufacturing process. The load distribution along the spline engagement length is also non-uniform because of tooth misalignments and shaft torsional effects. This paper presents an investigation of the influence of spur gear loads on the load distribution of spline teeth.

Introduction

Spline couplings are often used in power transfer systems to connect mechanical components such as shafts, flanges, brakes, clutches, pulleys, sprockets, and gears. A spline coupling has multiple teeth equally spaced around its circumference, which results in higher load capacity than a conventional single key. Spline teeth can be straight sided, in which both tooth flanks are parallel to each other with the same tooth thickness along the tooth height. Involute profiles are also used in spline teeth. Involute spline teeth are similar to gear teeth but shorter in height to provide great strength and compact size. Involute splines are typically preferred over parallel-side splines because they best center the two connecting components radially, and also provide lower root stresses with a larger tooth base thickness and smooth transition from tooth side to fillet radius. In a typical involute spline coupling of a shaft-gear connection, the shaft has the external teeth machined on it in the same number of internal grooves machined at the gear bore.

Ideally both the external teeth and internal grooves should have the same size to result in no clearance between them. Perfect splines under no clearance condition would evenly share the total load among the spline teeth in the circumferential direction. However, real-life splines are commonly designed to have a certain amount of allowable clearance on tooth sides and diameters to make them easy to assemble, to accommodate manufacturing tolerances, and also to allow lubricant to flow through the splines to help prevent fretting-type wear (Ref. 1).

Depending on the application, the spline fit is defined on tooth sides, between major diameters, or between minor diameters of the splines. Diameter-fits are used in applications where reduced radial clearance is required. In those cases the spline diameters are hard finished after heat treatment to a tight tolerance (Ref. 2). On the other hand, side-fit splines are often soft machined only, with no additional post-heat treatment operation, which provides a cost advantage over diameter-fit splines. The downside is larger variation among parts and larger radial clearance. The side clearance causes non-linearity similar to other components such as gears, bearings, and clutches, which, when combined with manufacturing deviations, such as spacing errors, and heat treatment distortions, result in uneven load sharing among spline teeth, especially in the circumferential direction, with consequent stress increase (Refs. 3-4).

Analytical and experimental studies done by Tjernberg (Ref. 3) showed that about half of the spline teeth carry load because of spacing errors, resulting in between 26% to 36% stress increase and over 50% life reduction. Chaplin (Ref. 1) also recommended assuming that half of the teeth share the full load. When subject to torsional load, splines demonstrate non-uniform load distribution along the engagement length of the tooth, which is in the axial direction, because of shaft torsional effects (Fig. 1) (Refs. 5-7). Volfson (Ref. 6) suggested that about a quarter of the teeth carry the full load. More recently, Chase (Refs. 8-9) presented a statistical approach to determine the load distribution in a spline coupling, and showed for a 10-tooth spline case study that approximately half of the teeth carried the full load.

It becomes clear from previous studies published in the literature that both manufacturing deviations, especially spac-

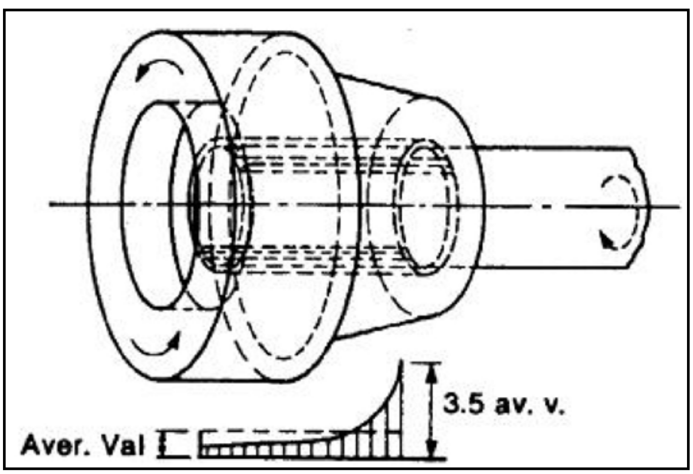

Figure 1 Load distribution in the axial direction for a pure circumferential torsional load case (Ref. 6).

ing errors, and shaft torsion significantly affect spline load distribution in the circumferential direction and in the axial direction. However, the studies were limited to the pure torsion loading condition only. In the particular case of drive train applications where involute splines are often used to connect gears to shafts, the gear mesh loads cause the splined components to be offset from their common center axis, affecting the load distribution of the spline teeth.

The objective of this paper is to investigate the gear load effects on spline load distribution, and propose a generalized and practical technique that can be used in the gear industry to calculate spline load distribution and to determine spline load capacity. A parametric procedure was developed to determine the gaps between internal and external spline teeth, accounting for the gear mesh loads and manufacturing deviations such as spacing errors. The general formulation of the problem of load distribution in gear teeth was applied to splines. The generalized elastic contact problem was solved using a simple procedure given in AGMA 927 (Ref. 10) and ISO 6336-1 (Ref. 11). Elastic deflections of spline teeth were calculated using a constant stiffness value that was obtained through Finite Element Analysis (FEA) of a spline coupling model in Reference 8.

The results showed that gear mesh loads significantly affect the load distribution of spline teeth. The maximum spline tooth load increased as the amount of side clearance between the internal and external splines increased. The procedure only applies to side-fit splines that have sufficient radial clearance between major diameters and minor diameters. Although not as accurate as finite element analysis (FEA), the spreadsheet solution is ideal for the beginning design work because it is fast and does not require FEA capability.

Analytical Model

A simple iterative method for the load distribution evaluation of spline teeth was developed from initial gaps among the spline teeth. Manufacturing deviations such as spacing errors and misalignment in the axial direction, intentional design modifications such as lead crown, and the spline centers offset by the gear loads were considered. When subject to gear loads, the pair of mating spline teeth with the smallest gap comes into contact first and load is transferred through it, resulting in elastic tooth deflections. Those deflections cause the gaps between the other spline teeth to get smaller and eventually other pairs of teeth come into contact and share the load. The final number of teeth in contact and their load sharing depend on the gap distribution, the elastic deflections and the load applied. An iterative procedure based on AGMA 927 (Ref. 10) and ISO 6336-1 (Ref. 11) was used to solve the load distribution problem, which was implemented into a spreadsheet. In the first step of the computational process, the spline teeth were divided into n points — or stations — across the engagement length. The gear loads were calculated from the gear tooth geometry and torque transferred through the gear mesh. The initial gaps were calculated from the spline geometry and radial location of one component to the other. The gaps also included manufacturing deviations and tooth misalignment. Then the load was evenly spread at all stations of all teeth to calculate the initial spline tooth elastic deflections such as bending and torsion. The total displacement of each point was obtained by adding the elastic deflections to the initial gaps under no load. From that point onward an iterative procedure was used to identify the non-contacting points and adjust the load values. The equations, assumptions and other details of the method are described in the following sections.

Gaps Analysis and Tooth Engagement

In perfect splines with no manufacturing deviations, no assembly misalignments, and both splined components sharing a common center axis, the side clearance is equal to the difference between the circular space width of the internal spline grooves and the circular tooth thickness of the external spline teeth taken at the same diameter (Fig. 2). The side clearance is the same for all pairs of mating spline teeth around the coupling, and also across the spline length. When transmitting torque through the coupling, the splined driver component turns in a given direction to eventually close the gap on the drive flanks. In the case of perfect splines, all mating spline teeth come into contact simultaneously and share the full load evenly in the circumferential direction.

One important manufacturing deviation of spline teeth for load distribution is spacing error (Refs. 3, 8). Spacing errors cause the spline teeth to be misplaced in the circumferential direction related to their theoretical location. This means

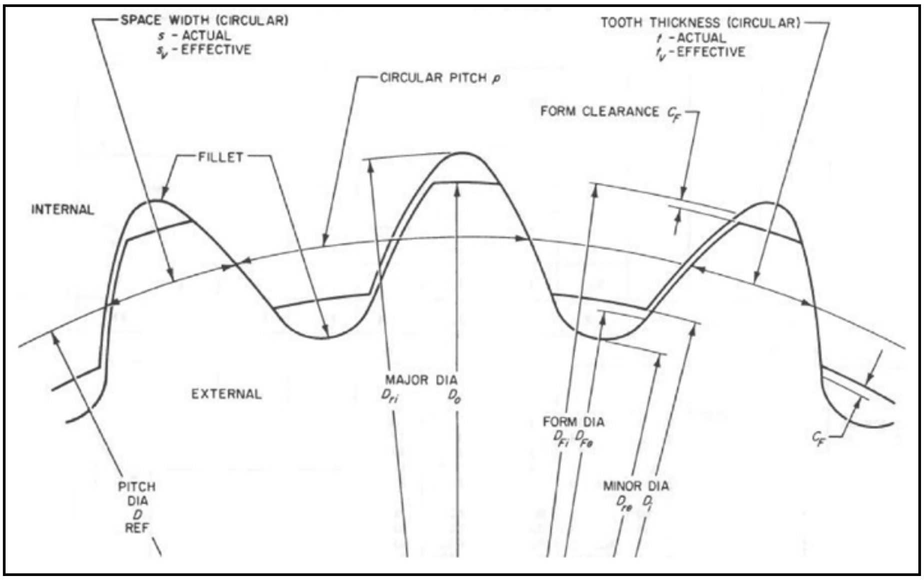

Figure 2 Side clearance of perfect splines (Ref. 12).

the teeth are not equally spaced around the circumference, which results in different gaps (Fig. 3). Spacing errors were entered in this model as a circular value with positive sign to indicate more gap (opposite to what is shown in Figure 3), and negative sign to indicate less gap (Fig. 3). The spacing errors of internal and external spline teeth were added together for a given assembly position, which defines the mating pairs of external teeth and internal groove with their respective spacing errors. The worst case for analysis occurs when the external tooth of maximum spacing error is assembled with the internal groove also with maximum spacing error but in the opposite direction.

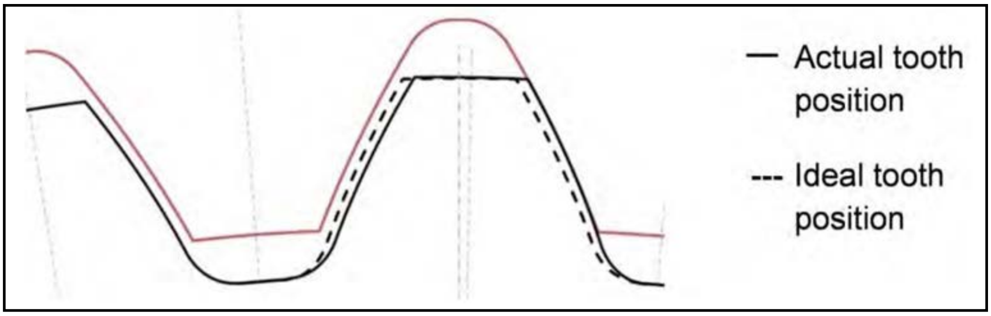

Figure 3 Spacing error of a spline tooth.

Misalignments and linear modifications in the tooth axial direction were also entered as a combination of deviations of the internal and external splines. Positive sign is used to indicate that the gap starts as zero at the left-hand end of the spline coupling and increases linearly towards the right-hand side of the coupling.

Lead crown is a barrel-shape modification across the length of the splines, where its maximum value is found at both ends of the spline teeth. Material is removed from the spline teeth according to a quadratic function from the middle of spline length toward the spline ends. In the model, lead crown was considered for the external splines only. It would be of small practical relevance to consider lead crown for internal splines based on manufacturing difficulty using conventional processes.

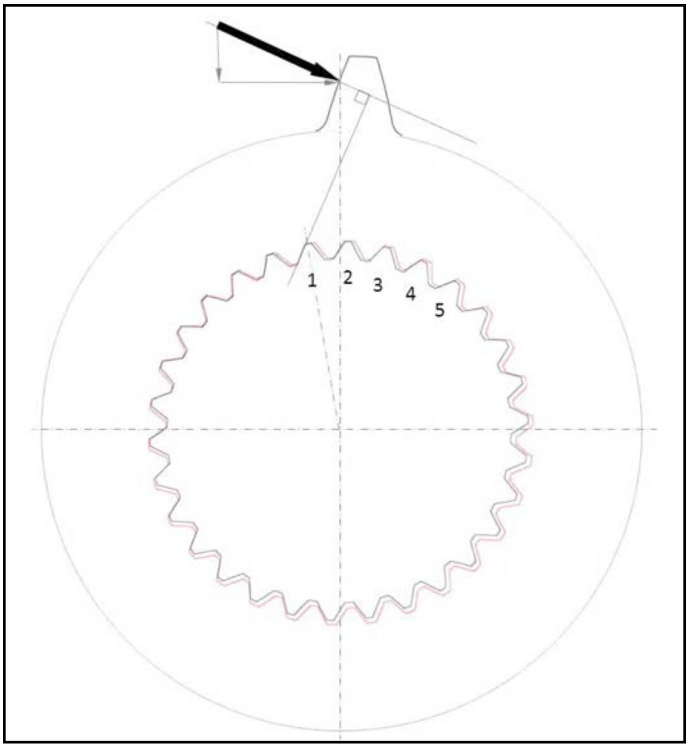

Figure 4 Gear load effect on gap distribution.

All deviations and modifications were calculated at the pitch circle diameter to each station and tooth, and were summed as gaps. Involute profile errors were neglected in the model because they are typically small compared to other factors. The gaps were assumed to be constant along the tooth height.

When the spline coupling is subjected to external loads other than pure torsion, such as gear loads, the gap distribution changes because the center of one component moves away from the center of the other component. This causes the gaps to vary around the spline-coupling circumference. Figure 4 shows an example of the effects of external load from a spur gear on gap distribution. In that case the shaft (external splines) is kept in place, and the gear (internal splines) moves in the line-of-action direction where the gear load is normal to the tooth flank. The gap variation among the pairs of spline teeth increases as the center displacement increases. The maximum displacement of the spline center related to its initial position depends on the geometrical relationship of spline teeth to the line-of-action angle. The worst-case condition was observed when the drive flank of a pair of spline teeth is perpendicular to the line-of-action (Fig. 4). Spline tooth No. 1 enters into contact first when no spline errors are considered. The gaps increase for the spline teeth farther from tooth No. 1.

The center distance displacement to bring the tooth number 1 into contact was calculated using the following Equation 1.

$$C = es \cos \phi_s \tag{1}$$

where

C is the distance from the center of the external splines to the center of the internal splines when an external load is applied;

es is half of the clearance between the circular space width of the internal grooves and the circular tooth thickness;

ϕ_s is the spline tooth pressure angle at the pitch circle diameter.

From the center displacement calculated in Equation 1 the resultant gap at each pair of spline teeth was calculated by Equation 2.

$$hc_i = es - C\frac{\cos\theta_i}{\cos\phi_s} \text{ or } hc_i = es\,(1-\cos\theta_i) \quad (2)$$

where

hc_i is the additional gap between the internal spline groove and external spline tooth i because of the center displacement;

θ_i is the angular distance from the tooth No. 1 to the tooth I g given by $\theta_i = (i-1)\,2\pi/N$;

N is the number of spline teeth;

$i = 1, 2, \dots N$.

The total initial gap is the summation of gaps caused by position and form deviations of spline teeth and grooves, misalignments, and the additional gap when the spline centers are displaced from one to another in the direction of the normal gear load applied to the coupling. The gap of each spline tooth pair was calculated by Equation 3.

$$h_{ij} = hm_{ij} + hr_j + hs_{ij} + hc_i \quad (3)$$

where

h is the total initial gap without deflections, hm is the gap caused by the combined misalignment and modification in the tooth axial direction of external spline tooth and internal spline groove;

hr is the gap caused by lead crown of the external spline teeth;

hs is the gap caused by combined spacing errors of the internal and external splines;

hc is the gap because of center displacement of the internal and the external splines;

i, j are the tooth number and the station number, respectively, $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, n$ for n stations.

When load is applied, the tooth engagement follows a sequence based on the gap distribution. The pair of teeth with zero clearance, as the tooth No. 1 of Figure 4, engages first and begins to transmit the torque load. That tooth pair deflects with load until the next pair of teeth engages. At that point two pairs of teeth are carrying different amounts of load, and deflect enough to bring another pair of teeth into contact. The third pair of teeth begins to share load, and also deflects. This process of sequential engagement continues with increasing load until the full load is applied. In the end, the number of pairs of teeth sharing load will depend on the gap distribution and the full load. The amount of load on each pair of teeth is a function of the gap distribution. The most loaded pair of teeth is the first one to enter into contact and is the most likely to fail (Refs. 6, 9).

Spline Tooth Stiffness and Elastic Deflections

Torsional and bending tooth deflections were considered in this study. Hertzian contact deflections were assumed small for spline teeth because of the conformal-type shape of the involute spline teeth in which both external teeth and internal grooves have the same radius of curvature. Thus, no special contact deflection calculation was considered. It was assumed that the elastic deformations are small compared to the size of splines such that the surface curvatures of splines over the contact zone remain unchanged. The supporting shaft was assumed to be solid or hollow cylinders when computing torsional deflection. The gear body was assumed to be rigid. Friction effects between engaged spline tooth pairs were not recorded.

Each spline tooth was divided lengthwise into n stations. Tooth elastic deflections were combined to produce a single stiffness constant of a spline tooth pair, $C_{\gamma m}$, which includes the stiffness of the internal and external teeth. The stiffness at each station was assumed to be an independent spring. The springs are added together when multiple stations and multiple pairs of teeth are in contact. Equation 4 was used to calculate the tooth deflections at each contact point.

$$\delta t_{ij} = L\delta_{ij}\,C_{\gamma m}^{-1} \quad (4)$$

where

δt_{ij} is the deflection of tooth i at Station j, μm;

$L\delta_{ij}$ is load intensity, N/mm;

$C_{\gamma m}$ is the spline tooth constant stiffness, N/mm/μm.

The torsional deflection was calculated over the engagement length of the splines, assuming a cylindrical shaft with circular cross section. The outside effective twist diameter of tooth section was defined as the root diameter plus 0.4 times the normal module (Refs. 10, 11). Shaft torsional deflection at each station across the spline length was calculated using the following equation that is given in (Refs. 10 and 11).

$$\delta t_j = \frac{4d^2 \times 10^3}{G\pi\,(d^4 - d_{in}^4)}\left(\sum_{k=1}^{j} L_k\right)\left(\sum_{k=1}^{j-1} X_k\right) \quad (5)$$

where

δs_j is the torsional deflection at Station j, μm;

d is the effective twist diameter, mm;

G is the shear modulus;

d_{in} is the inside diameter, mm;

L_k is the load at Stations k, N;

X_k is the distance between adjacent stations, mm;

j is the load at station number, $j = 1, 2, \dots n$ for n stations.

Solution of the load distribution problem. The method described in References 10 and 11 for the solution of the load distribution problem of gear teeth was applied for splines. The method uses the concept of constant tooth stiffness to calculate the load sharing at each station from the overall gap and total load. Overall gap is given by the summation of initial gaps (Equation 3) plus the elastic deflections (Equations 4 and 5). The overall gap is given by Equation 6.

$$\delta_{ij} = h_{ij} + \delta s_{ij} + \delta t_{ij} \quad (6)$$

A pair of spline teeth comes into contact when one of the splined components moves in relation to the other, thus eliminating the gap between the teeth. The pair of teeth with the smallest gap is the first to enter into contact. Using that point as zero, relative gaps can be calculated to all other points related to that point zero. The relative gaps are then defined by Equation 7.

$$\delta r_{ij} = \delta_{ij} - \min(\delta_{ij}) \quad (7)$$

where

$min\,(\delta_{ij})$ is minimum gap among the stations of all teeth, which is the gap of the closest point of a pair of spline teeth.

The load intensity at each tooth and station is a linear function of the gap and the stiffness, and is obtained by rewriting Equations 4 and 7 into Equation 8.

$$L\delta_{ij} = \delta r_{ij}\, C_{\gamma m} \tag{8}$$

The total load F_g is the summation of the loads applied to all teeth and stations given by Equation 9.

$$F_g = \sum_{k=1}^{N} \sum_{j=1}^{n} L_{ij} \tag{9}$$

where
L_{ij} is the load at a point j of tooth i, N; $L_{ij} = L\delta_{ij} X_j$.

When the torque is applied to the spline coupling through a spur gear pair, the gear mesh load normal to the gear tooth, which is on the plane of action, pushes the internal splines against the shaft external splines in the same direction of the normal load. The normal load can be split into two components based on the gear tooth geometry — i.e., a tangential load and a radial load. The tangential load is applied in the circumferential direction tangent to the spline pitch diameter and is given by Equation 10.

$$F_t = \frac{2T}{d} \tag{10}$$

where
T is the torque applied to the gear that is connected to the splined shaft, Nm;
d is the spline pitch diameter, m.

The normal load pushes the splines down (Fig. 4) and reacts on some of the spline flanks, depending on their position related to the gear load. The gear radial load is given by Equation 11.

$$F_r = \frac{2T}{D_w} \tan \phi_w \tag{11}$$

where
D_w is the gear operating pitch diameter, m;
ϕ_w is the gear operating pressure angle, degrees.

A geometrical factor was developed to calculate the gear radial load component to the spline tooth center line, which is based on the spline tooth geometry and the position of the gear radial load direction to each spline tooth center line. The geometrical factor for the gear radial load is given in Equation 12.

$$a_i = \max\left\{0, \left[\frac{2\pi}{N}(\mathrm{N} - i + 1) + \frac{\pi}{2} - 2\phi_w + \phi_s\right]\cos\phi_s\right. \tag{12}$$

where
a_i is the radial load geometrical factor of spline tooth i;
N is the number of spline teeth;
ϕ_s is the spline tooth pressure angle at pitch diameter, degrees;
ϕ_w is the gear operating pressure angle, degrees.

In Equation 12, the factor a_i must be greater or equal to zero to signify load applied to drive spline flank in the direction of rotation; that is the reason for the maximum value condition.

The total load, F_g, applied to the spline teeth, is equaled to the summation of both gear load components, tangential and radial loads.

The load at the first station of the first tooth in contact is calculated from the gap distribution and total load using Equation 13, whose derivation can be found in References 10 and 11.

$$L_{11} = \frac{1}{nN}\left[F_g + C_{\gamma m} X_j \left(n\,N\,\delta r_{11} - \sum_{k=1}^{N}\sum_{j=1}^{n} \delta r_{11}\right)\right] \tag{13}$$

where
δr_{11} is the relative gap at Tooth No. 1 and Station No. 1;
δr_{ij} is the relative gap at Tooth i and Station j.

The difference in load between any two points is proportional to the difference in the relative gap between them. Thus, once the load at Tooth 1/Station 1 is obtained, the load at other points can be calculated by Equation 14.

$$L_{ij} = L_{11} + (\delta r_{11} - \delta r_{ij})\, C_{\gamma m} X_j \tag{14}$$

It is evident from examining Equation 14 that areas with greater gaps have lower load and areas with lower gaps have higher loads. It is also possible from Equation 14 to obtain negative results when the difference in gaps is large, which means that a specific point is not in contact. In that case the load at those points must be set to zero, and the loads at the contacting points need to be adjusted in a way that the summation of all load points equals to the total load as was given in Equation 9. After that, the adjusted loads are used to recalculate the deflections of each station of each tooth, and the gap analysis is updated. The loads can be re-calculated by Equations 13 and 14 with the new gap distribution. The load results are adjusted again if there are points with negative values. This iterative process continues until the difference between the gaps of two consecutive iterations falls within an acceptable error. Less than 3 mm is suggested in (Refs. 10-11) as a convergence criterion, which is usually achieved after a few iterations.

The final load distribution is the last iteration. A load distribution factor K_H is calculated for each spline tooth as the ratio of the total load at each tooth and the average tooth load, as shown in Equation 15 (Refs. 10-11).

$$K_{Hi} \frac{L_i}{L_{avg}} = \frac{N}{F_g} \sum_{j=1}^{n} L_{ij} \tag{15}$$

where
K_{Hi} is the load distribution factor of Tooth i;
L_i is the total load applied on Tooth i, which is the summation of loads applied to n stations across the tooth engagement length;
L_{avg} is the average load, which is the total load transmitted through the spline coupling divided by the number of teeth, N.

A load distribution factor equal to one means the load is evenly shared among all spline teeth. A factor greater than one means Tooth i is carrying more load because of non-uniform load-sharing among the spline teeth.

In a similar way, an axial load distribution factor across the spline tooth length can be calculated as the ratio of the load applied at a station and the average load applied to the stations of that tooth.

$$K_{Aij}\frac{L_{ij}}{L_{avg\,i}} = L_{ij}\,n\left(\sum_{j=1}^{n} L_{ij}\right)^{-1} \tag{16}$$

where
K_{aij} is the axial load distribution factor of tooth i, at station j;
n is the number of stations across the tooth engagement length;
$L_{avg\,i}$ is the average load applied to Tooth i.

The load distribution factor, K_H, can be applied to calculate spline tooth stresses, such as compressive, shear and bending stress, and to determine the spline coupling torque rating. The axial load distribution factor, K_A, can be helpful to assess effects of misalignments, shaft torsional effects — especially for long spline teeth — and to determine the need to use lead modifications such as lead crown.

Results

The proposed procedure to calculate load distribution of spline teeth was implemented into a spreadsheet and used to evaluate the effects of gear loads of a spur gear pair on the load distribution of spline teeth.

Example problem definition. In the study case, a spline coupling that connects a spur gear with internal splines to a splined shaft was investigated as a numerical example. Torque was transmitted from the gear teeth to the output shaft through the spline coupling. The internal and external spline data is shown in Table 1. The spline engagement length was 30 mm. The spline tooth constant stiffness was assumed as 16 N/mm/μm (Ref. 8). The spline teeth were divided into 18 stations, as recommended (Ref. 11) for gear teeth. Both the gear and the splined shaft were made of steel (modulus of elasticity 2.06 x 10 N/mm^2 and shear modulus 8.3 x 10^4 N/mm^2). The gear teeth and gear body were assumed to be rigid. The spur gear pair used for the case study was a 30-tooth gear pair, 1:1 gear ratio, 4 mm module, 20° pressure angle, and 120 mm center distance. Torque applied to the gear was 3000 Nm.

Tooth load distribution for perfect splines subject to gear loads. The study was done initially assuming perfect splines, which means no misalignments, spacing errors, or other errors, and no intentional modification.

The gap analysis was done using Equations 1 and 2 for different levels of clearances between the circular space width of the internal grooves and the circular tooth thickness according to the tolerances of Table 1. The corresponding load distribution problems were solved to determine the load applied to each tooth and as well as the load distribution factors. Figure 5 shows the results for 3,000 Nm torque that was applied through the gear to the spline coupling, and 0.050 mm side clearance, which resulted in 0.022 mm displacement of the internal spline center to the external spline center.

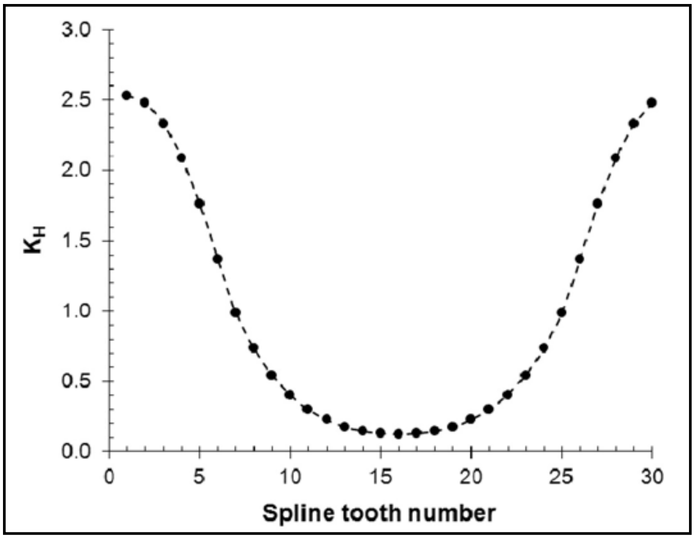

Figure 5 Tooth load distribution factor for 3,000 Nm torque and 0.050 mm side clearance.

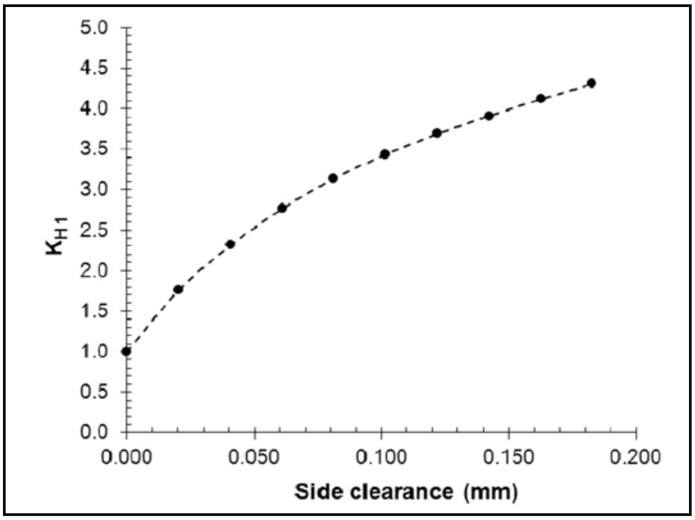

Figure 6 Relationship between maximum tooth load and side clearance.

Table 1 Example: Spline data (ANSI B92.1-1970) (Ref. 12)

Internal involute spline data		External involute spline data	
Fillet root side fit	Tolerance class -5	Fillet root side fit	Tolerance class -5
Number of teeth	30	Number of teeth	30
Spline pitch	16/32	Spline pitch	16/32
Pressure angle, degrees	30°	Pressure angle, degrees	30°
Base diameter, mm	41.2445 ref.	Base diameter, mm	41.2445 ref.
Pitch diameter, mm	47.6250 ref.	Pitch diameter, mm	47.6250 ref.
Major diameter, mm	50.90 max.	Major diameter, mm	49.12/49.07
Form diameter, mm	49.33	Form diameter, mm	45.92
Minor diameter, mm	46.18/46.05	Minor diameter, mm	44.02 min.
Circular space width		Circular tooth thickness	
Actual, mm	2.570/2.537	Maximum effective, mm	2.494
Minimum effective, mm	2.494	Actual, mm	2.421/2.388

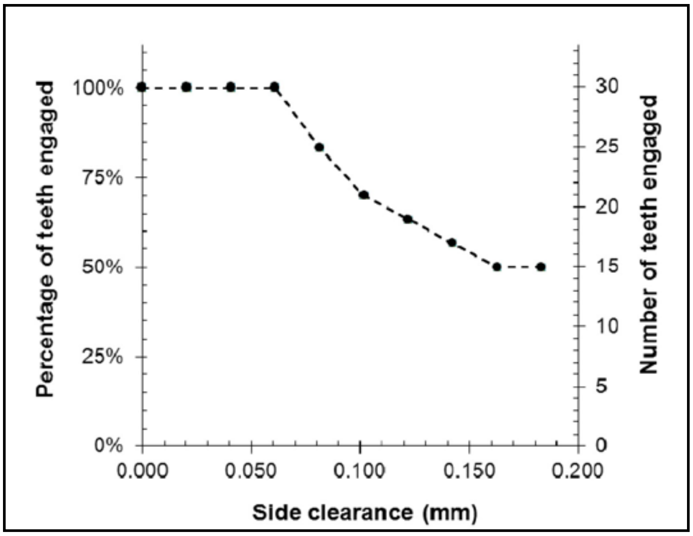

Figure 7 Number of teeth in contact, based on side clearance for perfect splines.

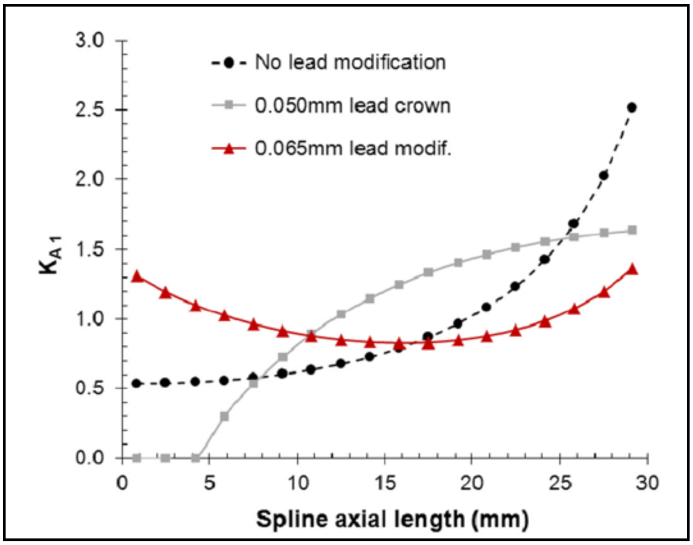

Figure 8 Axial load factor without and with lead modifications.

The results of Figure 5 show that the first pair of spline teeth to engage, which is Tooth No. 1 and with the smallest gap, carried 2.53 more load compared to the average load (uniform load distribution). Tooth No. 16, which is diametrically opposite to Tooth No. 1 and had the largest gap, carries only 12% of the average load ($K_{H16} = 0.12$).

Tooth side clearance effect on spline maximum load distribution factor. The maximum load distribution factor K_{H1} changed as a function of the side clearance, which depends on the dimension of both internal and external splines. Figure 6 shows the maximum load distribution factor results, which were found on Tooth No. 1, for different side clearance values. It was observed a significant effect of side clearance on K_{H1} results.

As expected, the worst case was found at the maximum side clearance, which occurs when the actual circular space width is in the upper limit and the actual circular tooth thickness is in the lower limit. At that condition the maximum load distribution factor K_{H1} was 4.31. As the side clearance decreases, K_{H1} goes down towards the value of 1, which means uniform load distribution among the spline teeth.

The overall stiffness of the spline coupling depends on the number of pair of teeth engaged. Looking back to Figure 5, it can be observed that all teeth were engaged and carrying some amount of load. In other cases, with larger side clearance, some of the spline teeth were not in contact because of the large gap difference at the first tooth to engage. Figure 7 shows the percentage of teeth engaged for each side clearance level; 100% means that all 30 teeth were engaged and carrying some load. Under maximum side clearance, 50% of the spline teeth (or 15 teeth) were engaged and carrying some load.

Effect of tooth spacing error on maximum load distribution factor. Both manufacturing deviations and intentional modifications on spline teeth have a significant effect on spline load distribution — especially spacing errors (Refs. 3, 8). Repeating the analysis with a spacing error of 0.015 mm added to the pair of teeth labeled No. 1 for the same case study, with 0.050 mm side clearance, resulted in a 51% load increase on that pair of teeth. Tooth No. 1 carried more load until the next pair of teeth could engage to begin sharing the load. The load distribution factor increased from $K_{H1} = 2.53$ (no spacing error) to $K_{H1} = 3.83$ (0.015 mm spacing error).

Effects of lead modifications on axial load factor. In the tooth axial direction, the shaft torsional effect was observed as a significant parameter that affects the load distribution across the spline tooth engagement length. The axial load distribution factors were determined in the case study with 0.050 mm side clearance. The maximum factor K_{A1} was 2.51, which is in line with previous studies in the literature (Refs. 4, 6). On Tooth No. 1 the axial load distribution factor varied from 0.54 at one tooth end to 2.51 at the opposite end, which is the tooth end of torque reaction. Lead crown and lead modification were then applied to the spline teeth to investigate their effects on the axial load distribution factor. Figure 8 shows the results without any axial modification, with 0.050 mm lead crown and 0.065 mm lead modification.

The results of Figure 8 show that lead crown reduces the high loads at the end of the tooth, but increases load in the middle of the spline tooth. Moreover, three stations on the free end of the spline teeth carried no load. On the other hand, the lead modification produced a more uniform load distribution across the stations, in the axial direction of the tooth. That means about a 46% reduction on maximum tooth load in the axial direction. **PTE**

Discussions

- A simple model was developed for load distribution analysis of splines that are subjected to spur gear mesh loads and side-fit splines that have sufficient radial clearance between both major and minor diameters.
- A gap analysis was performed, based on the spline tooth dimensions and direction of gear normal load.
- The gear mesh load was calculated and used to determine the sharing among the spline teeth that were divided axially into stations.
- A constant for tooth stiffness was used to calculate tooth deflections.
- The load distribution problem was solved using a simple approach from industry standards for load distribution in gear teeth. The method was implemented into a spreadsheet and used to investigate the load distribution of a spline coupling to connect a spur gear to a splined shaft.
- The results showed a significant effect of side clearance on the load-sharing among the spline teeth for side fit-type splines.
- The case study results — including spacing errors and lead modifications — were in line with previous studies in the literature.
- On spline couplings used to connect gears to shafts, the gear loads — such as radial and tangential loads — add an extra increase to the load factor due to the gap distribution changes caused by the center displacement of one splined component to the other. This is a function of the side clearance.
- The circular space width and circular tooth thickness, which define the side clearance range, are critical parameters for the load distribution factor of spline couplings in gear applications. Those dimensions and tolerances are defined according to the tolerance class and fit class of industry standards. The numerical example used in this study was a tolerance class 5 and fit class h spline. Splines with tolerance classes and fit classes greater than class 5 are expected to present an even larger increase on maximum tooth load and, consequently, higher stresses.
- The simple method incorporated into the spreadsheet could be helpful to designers to quickly assess load distribution of spline teeth in gear applications, determine tooth stresses, and define lead modifications as needed.
- As future work, the model could be extended for helical gears by including two additional momentums of the helix angle.

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