A New Geometrically Adaptive Approach for Tooth Contact Analysis of Gear Drives

Francisco Sanchez-Marin, Alfonso Fuentes, Jose L. Iserte and Ignacio Gonzalez-Perez

Tooth contact analysis (TCA) is an important tool directed to the determination of contact patterns, contact paths, and transmission errors in gear drives. In this work, a new general approach that is applicable to any kind of gear geometry is proposed.

Introduction

Tooth contact analysis is a tool to simulate the meshing of gear drives being a very important resource to predict its performance. The main hypotheses of the TCA are that the transmission is considered unloaded and that the tooth contact surfaces of the gears are rigid. On the other hand, the two main results of the TCA are the contact pattern of the gearset and the function of transmission errors that represents the performance of the gearset.

The first works of development of TCA were done by Litvin and Kai (Refs. 1–2), and Baxter (Ref. 3), followed by the engineers of Gleason Works (Ref. 4). From then, a number of researchers proposed new approaches with different objectives encouraged by the increasing computation power of modern computers.

In more recent works, Vogel et al (Ref. 5) proposed a constructive approach for the contact analysis of hypoid bevel gears where the paths of contact, the transmission error and the contact ellipses are obtained directly by modelling the underlying virtual machine tool and its derivatives. Litvin et al (Ref. 6) developed a numeric approach for tooth contact analysis including the automatic determination of the guess values for derivation of the first contact point of tooth surfaces. Vecchiato (Ref. 7) extended the principles of TCA for the simulation of meshing of a planetary gear drive with a set of planets. Simon (Ref. 8) proposed a method for computer aided tooth contact analysis in spiral bevel gears for the investigation of the influence of machine tool settings on the path of contact, potential contact lines, separations along these lines and on angular position error of the driven gear. Bracci et al (Ref. 9) presented a geometric approach to the estimation of the contact pattern of a hypoid gear drive where tooth, gear body, shaft and housing deformations are approximately taken into account by properly selecting the marking compound thickness and topography. Sobolewski et al (Ref. 10) proposed an approach for tooth contact analysis based on the use of a CAD environment for spiral bevel gearsets with tooth flanks represented as CAD free-form surfaces.

In this work, a new approach for unloaded TCA is proposed with the following objectives:

- must be general and applicable to any type of gear, including types with line contact and types with point contact
- must work properly for any relative position of the tooth contact surfaces, including both aligned and misaligned (in any way) relative positions
- final accuracy must be parameter-dependent, to be able to be controlled by the client of the approach
- computational cost must be as low as possible

The proposed method is a geometric approach based on the discretization of the tooth contact surfaces and the progressive adaptive refinement of the obtained meshes to solve the contact problem and to compute the instantaneous contact area for each position of the gearset along the gearing cycle. The approach requires the existence of a mathematical model (in the form of a parametric function) of the contact surfaces that involves the solution of the gearing Equation 11. It also requires the existence of an algorithm to obtain a basic triangle mesh of the tooth contact surface, including determination of the tooth contact surface limits based on the defining parameters of the gear.

Algorithm to Compute the TCA

The TCA algorithm has the objective of solving the contact problem for different positions of the gearset along the gearing cycle and computing the transmission error and instantaneous contact area for each position. The angular position of the gearset is defined here by the angular position of the pinion and the final angular position of the wheel is the one that brings the wheel in contact with the pinion.

The first part of the algorithm solves the positional contact problem for a number of equally separated positions of the gearing cycle. The number of positions is specified by the angular increment of the pinion that is computed as the pinion step angle divided by a number. The detection of the limits of the gearing cycle for the reference teeth pair is computed by solving the positional contact problem for both the following and previous teeth pairs, and then determining which teeth pair contacts first for each specific position of the pinion.

This paper was originally presented at the 2014 International Gear Conference, Lyon Villeurbanne, France and is republished here with the authors' permission.

In the second part, the algorithm uses the angular position of the wheel to be in contact with the pinion, obtained in the first part. Then, for each position of the pinion (that is, for each position of the gearset), it computes the transmission error and the instantaneous contact area, ending the approach.

Algorithm to Solve Positional Contact Problem

This algorithm the solves tooth contact problem for a reference teeth pair (one pinion tooth contacting with one wheel tooth) and for a specific position of the gearset (positional problem). Thus the algorithm starts from a given angular position of the pinion tooth and an initial position of the wheel tooth (not necessarily in contact), and has the objective of computing the angle that is necessary to rotate the wheel from that position for the wheel tooth to contact the pinion tooth. The algorithm has been designed to be independent from the relative position of the gears, being suitable when the gears are aligned and when they are misaligned in any way.

The input parameters of the algorithm are:

- *Triangle refinement goal size*: the meshes are refined around the contact point until the size of the mesh triangles is under this value.
- *Tolerance for angle of rotation to contact the opposite mesh (or angular tolerance):* this value delimits the area of the contact surfaces that is going to be refined in each loop iteration (see step 3). This value is reduced from iteration to iteration to reduce the refinement area around the contact point.

The algorithm includes the following steps:

Step 1: The algorithm gets a basic triangle mesh of pinion and wheel contact surfaces. These meshes are moderately coarse and define the limits of the contact surface. The pinion mesh is at the pinion position of computation and the wheel mesh is at the initial reference position of the wheel.

Step 2: For each node of the wheel mesh, the angle of rotation around the wheel axis for the wheel node to contact the pinion mesh is computed. Reciprocally, for each node of the pinion mesh, the angle of rotation around the wheel axis for the pinion node to contact the wheel mesh is computed. Thus, each node has an associated value of angle of rotation to contact the opposite mesh.

Step 3: The angular tolerance is used to refine the wheel mesh. This way, all triangles having nodes with an associated value of angle of rotation to contact the pinion mesh under the tolerance of refinement are split adding a new node in the midpoint of the longest edge. The position of the new nodes is updated to be on the contact surface and its angle of rotation to contact the pinion mesh is computed.

Step 4: The angular tolerance of is used to refine the pinion mesh. The procedure is the same as it was explained in step 3 for the refinement of the wheel mesh.

Step 5: The algorithm checks if the required size of triangles has been reached to exit the loop. If the size of all refined triangles in pinion and wheel meshes is under the triangle goal size, the algorithm jumps to step 7, exiting the loop.

Step 6: The angular tolerance is reduced to decrease the mesh portion to be refined in next iteration and, then, the algorithm jumps to step 3.

Step 7: Compute the lowest angle of rotation of the wheel to contact the pinion mesh. The contact point on the pinion mesh and the contact point on the wheel mesh.

The results (output) of this algorithm are the angle that is necessary to rotate the wheel around its own axis for the wheel tooth surface to contact the pinion tooth surface, being the pinion at the specified position, and the contact points in both contact surfaces.

Algorithm to Compute the Instantaneous contact area

Given the pinion and wheel reference teeth pair in contact for a specific position of the pinion, this algorithm has the objective of obtaining the instantaneous contact area associated to a specific distance value: i.e., the virtual marking compound thickness.

The input parameter of the algorithm is:

- *Virtual marking compound thickness* (VMCT): it is the distance value that defines the instantaneous contact areas on both pinion and wheel teeth. Thus, the instantaneous contact areas on the pinion (resp. wheel) is composed by the points of the contact surface that are at a distance from the wheel (resp. pinion) surface that is equal to the VMCT.

The algorithm includes the following steps:

Step 1: the algorithm gets a basic mesh of pinion and wheel contact surfaces being the reference teeth pair in contact for the specified position of the pinion.

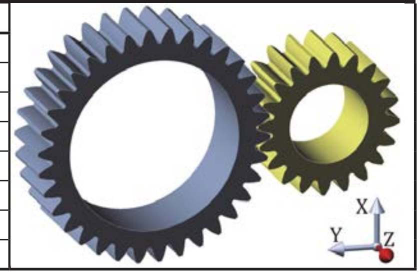
Step 2: for each node of the wheel mesh, the distance from the wheel node to the pinion mesh is computed. Reciprocally, for each node of the pinion mesh, the distance from the pinion node to the wheel mesh is computed. Therefore, each node of each mesh has an associated value of distance to the opposite mesh.

Step 3: the VMCT is used to refine the wheel mesh. This way, all triangles having nodes with associated over and under the VMCT are split adding a new node in the midpoint of the longest edge. The position of the new nodes is updated to be on the contact surface and its distance to the pinion mesh is computed.

Step 4: similar to step 3, the VMCT is used to refine the pinion mesh.

Table 1 Gearset data

	Pinion	Wheel
Type	Standard spur	
Module	1 mm	
Pressure angle	25 deg	
Num. of teeth	20	34
Addendum	1 mm	1 mm
Dedendum	1.25 mm	1.25 mm
Face width	10 mm	10 mm
Centre distance	Standard	



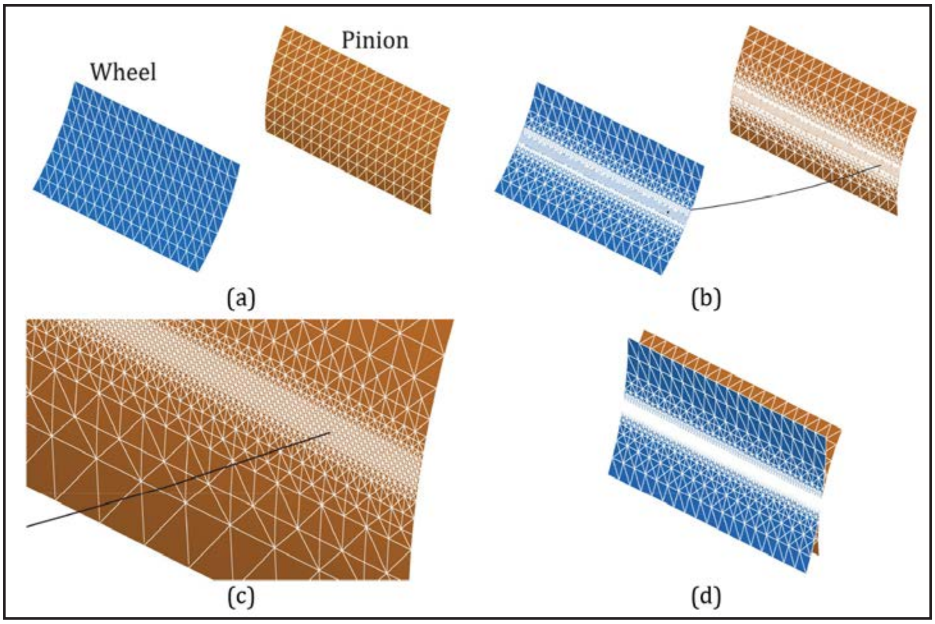


Figure 1 Results of the positional contact problem for case I.

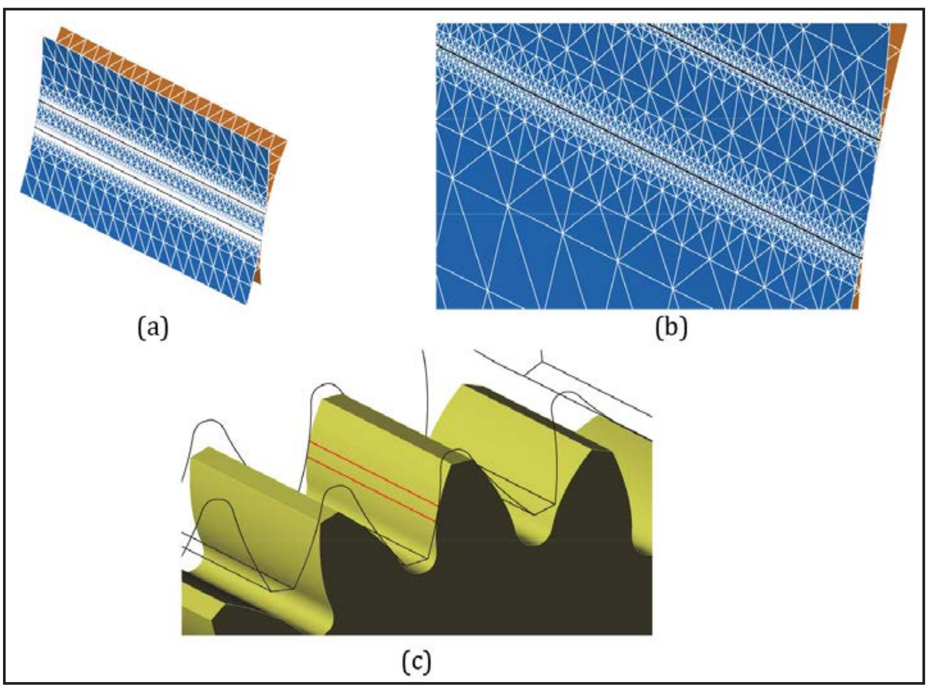


Figure 2 Computation of the bearing contact and transmission errors for case I.

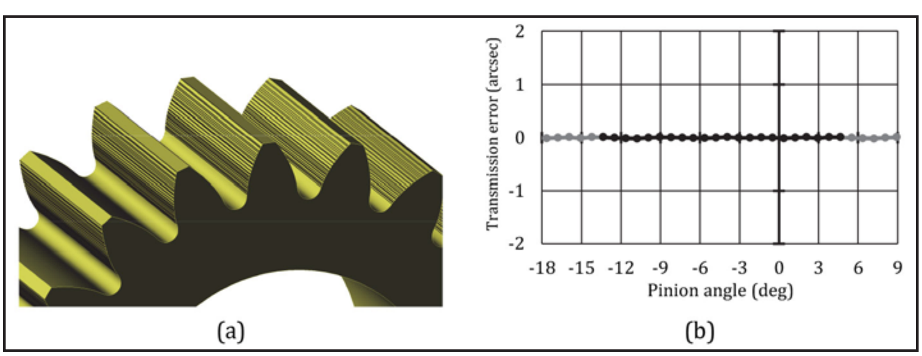


Figure 3 Final results of the TCA algorithm for case I.

Step 5: the algorithm checks if the required size of triangles has been reached to exit the loop. From the previously refined triangles, if there are triangles with size over the triangle goal size, the algorithm jumps to step 3. Otherwise, the algorithm continues with step 6.

Step 6: compute the instantaneous contact areas from the pinion mesh by linear interpolation from the distance values associated to the nodes. Similarly, computes the bearing contact curve from the wheel mesh. The results (output) of this algorithm are the instantaneous contact areas corresponding to the pinion and wheel teeth for the initial contact position of this reference teeth pair.

Test Cases and Discussion

The proposed approach has been tested in this work with a spur gearset. This type of transmission is a good test example because it has a line contact when the gearset is aligned, and that supposes an important difficulty for other TCA approaches. On the other hand, when the gears have an angular misalignment, there is a point contact and the bearing contact shifts to one of the edges, what makes it a good test for the approach as well.

The parameters of the used spur gearset are shown in Table 1. The global size of the gears is not relevant for the proposed approach, so an arbitrary value of the module has been selected. The rest of the parameters have been assigned with typical and normalized values.

Case I: standard spur aligned gearset. In the first test case, the spur gears are perfectly aligned. The TCA algorithm has been executed obtaining the contact pattern and the transmission errors along the gearing cycle. For each position of the gearing cycle, the algorithm solves first the positional contact problem and, then, computes the bearing contact. For both problems, a value of 0.05 mm has been used for the triangle refinement goal size. For the computation of the bearing contact, a value of 0.0065 mm has been used as virtual marking compound thickness (VMCT).

Given a specific position in the gearing cycle, the algorithm to solve the positional contact problem starts with a basic triangle mesh of pinion and wheel contact surfaces (Fig. 1a). After computing the contact with successive refinement of both meshes, the contact point and the angle that is necessary to rotate the wheel to contact the pinion is obtained (Fig. 1b). Figure 1c shows a detail of the pinion refinement to determine the contact point on the pinion surface. It can be observed how the area close to the contact line has been adaptively refined. Finally, Figure 1d shows both refined meshes in perfect rigid contact.

After solving the contact problem, the algorithm computes the instantaneous contact area starting with the same basic triangle mesh of pinion and wheel contact surfaces (Fig. 1a) but being the meshes at the previously obtained contact position. Then, the refinement iteration is performed and the resulting instantaneous contact areas for pinion and wheel are obtained (Fig. 2a). A detail of the refinement of the meshes to obtain the instantaneous contact areas is shown in Figure 2b. The final representation of the instantaneous contact area on the pinion is shown in Figure 2c. It can be observed that the algorithm predicts a instantaneous contact area according to the expected line contact.

Finally, Figure 3 shows the results of the TCA algorithm: contact pattern for the pinion (Fig. 3a) and the transmission error graph (Fig. 3b). Since the gears are perfectly aligned and the tooth geometry is standard (involute) for both gears, a full side contact pattern and a zero transmission error were expected. The graph shows in black the transmission errors for a pinion step angle (18° in this example) that corresponds to a gearing cycle.

Case II: standard spur misaligned gearset. In the second case, the wheel has been misaligned with respect to the pinion. The imposed misalignment consisted in the displacement of the wheel −0.5 mm along the *Z* axis and the rotation of the wheel 0.1° around the *X* axis (see axis in Fig. 1). Similarly to case I, Figure 4 shows the intermediate results of the algorithm to solve the positional contact problem associated to a specific position of the gearing cycle. It can be observed how the contact type is point contact due to the misalignment and how the algorithm refines the mesh adaptively to compute the

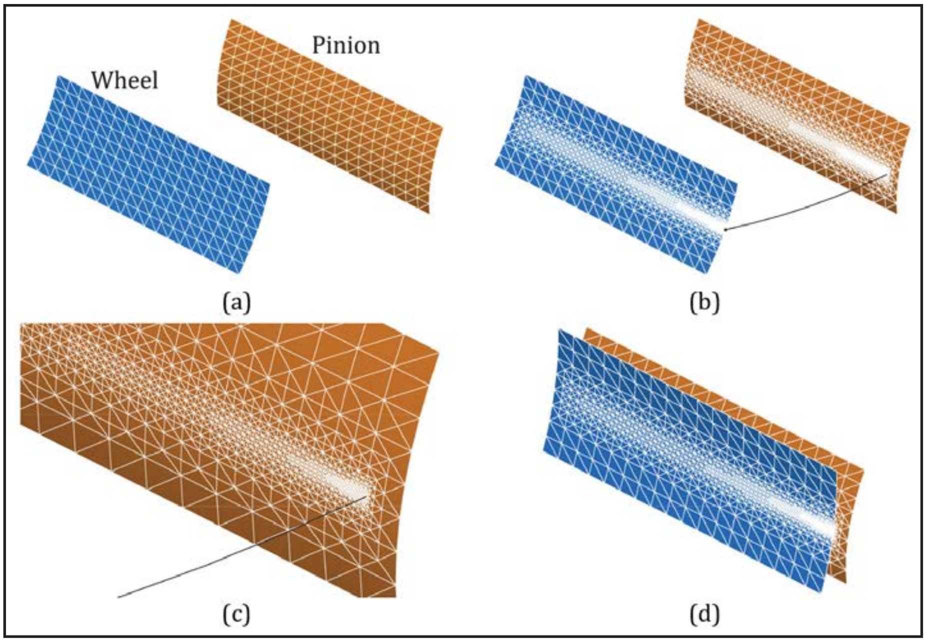


Figure 4 Results of the computation of the bearing contact for case II.

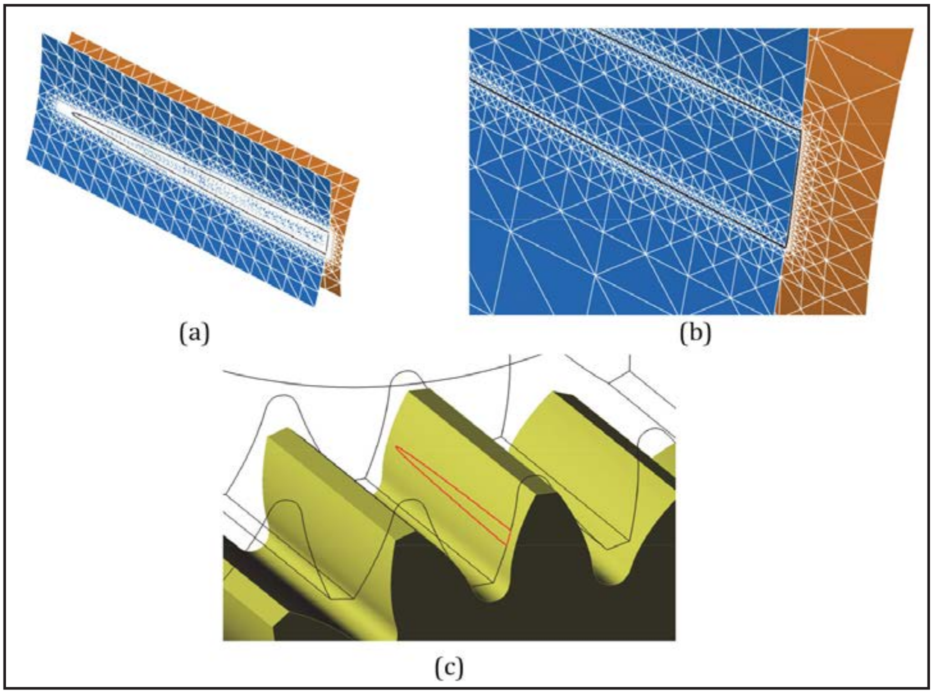


Figure 5 Results of the positional contact problem for case II.

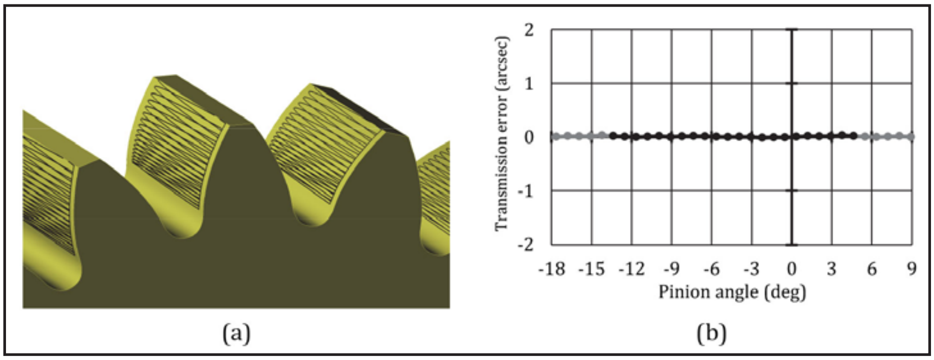


Figure 6 Final results of the TCA algorithm for case II.

contact point.

On the other hand, Figure 5 shows the intermediate results of the algorithm to compute the instantaneous contact areas. It can be observed that the adaptive refinement to compute the instantaneous contact areas associated to the value of the virtual marking compound thickness and the precision of the obtained curve in both meshes.

Finally, Figure 6 shows the results of the TCA algorithm. The obtained graph in Figure 6b indicates that the combination of the small face width with the small imposed angular misalignment does not generate significant transmission error along the gearing cycle.

Conclusions

In this work, a new geometrically adaptive geometric approach for the tooth contact analysis of gear drives has been proposed. The new approach solves the positional contact problem and the computation of the instantaneous contact area. The provided results of the approach are the transmission errors along the gearing cycle, the instantaneous contact area at any position of the gearing cycle and the contact pattern. The approach is general, independent of the gearset type and of the relative position of the gears, which makes it very versatile. The precision of the results is dependent on the degree of refinement that can be decided by the user.

The approach has been tested with two cases, one providing line contact and the other providing point contact due to misalignment and the approach has been demonstrated to adapt very well to the inherent geometric problem and to obtain very precise results in very low computational times.

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Francisco Sanchez-Marin - Associate Professor at the Department of Mechanical Engineering of the University Jaume I (Castellon, Spain) and Principal Investigator of the Gear Drives Research Group (GITE). MS in Mechanical Engineer by the Polytechnic University of Valencia (Spain, 1994). Product Manager in IBM (Valencia, Spain). Ph.D. in Mechanincal Engineering by the University Jaume I (Castellón, Spain, 2000). Author of different books and research papers on maintenance, biomechanical engineering, mechanism synthesis and gear transmissions. More than 19 years of teaching experience in mechanical engineering courses. Member of the Mechanical Engineering Spanish Association (AEIM).

Alfonso Fuentes - Principal Investigator of the Enhanced Gear Drives Research Group (GITAE). Mechanical Engineer (1993). Ph.D. in Mechanical Engineering (1996). Full Professor at the Polytechnic University of Cartagena at present. Author of more than 70 publications and 4 books. Involved in computerized development of improved gear transmissions applied in helicopters and automotive industry, development of enhanced computer models of gears for stress analysis and development of computer programs for generation and simulation of meshing of low-noise, stable bearing contact spiral bevel gears with improved geometry. Member of the Editorial Board of the Journal Mechanism and Machine Theory, the Open Mechanical Engineering Journal and the Recent Patents on Mechanical Engineering Journal. Member of the American Gear Manufacturers Association (AGMA) and the American Society of Mechanical Engineers (ASME).

Jose L. Iserte - Associate Professor at the Department of Mechanical Engineering and cofounding member of the Gear Drives Research Group of the Universitat Jaume I. Mechanical Engineer by the Polytechnic University of Valencia (Spain, 1997). Professional experience as a mechanical engineer in the Technical Division of the Astrophysics Institute of Canarias (Spain, 1999). Ph.D. by the Polytechnic University of Valencia (2005). Teaching experience in Machine Theory and Mechanisms. Member of the Mechanical Engineering Spanish Association (AEIM) since 2011.

Ignacio Gonzalez-Perez - Associate Professor at the Department of Mechanical Engineering of the Polytechnic University of Cartagena (Spain, 2009). Ph.D. by the Polytechnic University of Cartagena (Spain, 2003). Mechanical Engineer by the University of Murcia (1999). Author of more than 30 publications. Teaching experience in Machine Element Design for more than eight years. Research experience in Theory of Gearing and Applications since year 2001. Member of the American Gear Manufacturers Association (AGMA) and the American Society of Mechanical Engineers (ASME).