Determination of Load Distributions on Double Helical-Geared Planetary Gearboxes

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Standardized calculation methods such as ISO 6336 and DIN 3990 already exist to determine the load distributions on gears inside a planetary gearbox, but by their very universal nature, these methods offer varying results depending on the gearbox design. Double helical gears, in particular, can benefit from more specific, complex algorithms to reach a maximum level of efficiency. Double helical gears interact with the rest of the gearbox differently than helical or spur gears, and thus benefit from different analytical models outside the standardized methods. The present research project describes the algorithm to determine the load distribution of planetary gearboxes with double helical gears.

The state of the art

High efficiency by maximum usage of available space, high gear ratios, best possible efficiencies and a wide range of applications due to the possibility of switching operations, superposition and summation gearboxes are known properties of planetary gearboxes.

The optimization and effective utilization of planetary gearbox designs requires a detailed consideration of the loads on the gears in the gearbox. Therefore, standardized calculation methods such as ISO 6336 or DIN 3990 can be used. To benefit from the full potential of the teeth, while the safety of the teeth can be assured at all times, advanced analysis of load distribution will be necessary.

Because of the need for these calculation standards to be universally usable, these methods use approximations that cannot cover the special characteristics of planetary gearboxes and therefore cannot be used to optimize the gears to their full potential. For this reason, computer-aided calculation methods have been developed for planetary gearboxes with spur and helical gears that consider the most important influences on the load distribution, including housing deformation, bearing deformation, deformation of the planet carrier, deformation of the wheel bodies and the deformation of the teeth themselves. Using this information, a detailed load distribution is possible to reach the maximum capability of the gears.

Research project for double helical gears

It is of vital importance to interpret and use the single calculation results correctly. The different results are given by different methods in analytical and numerical calculations. Regarding a complex assembly like planetary gearboxes, there is an additional fact to consider by having all results for single elements, depending on the behavior of other elements, in the assembly. An iterative calculation algorithm is inevitable.

The need for high rotational speeds and torques in gearboxes leads to high axial loads and therefore high risks for the additional elements in the assembly — for example, the bearings. The solution for these axial loads can be done by using double helical gears where the axial load components compensate each other. Double helical gears, the resulting loads and the deformation of the wheel bodies, with possible influences from one gear side on the other, cause the assembly to have an entirely different behavior than planetary gear stages with spur or helical gears. Detailed load distributions cannot be calculated at the moment, because the results of the existing calculation algorithms are not regarding these specific characteristics, and approximations cannot be made at this point due to the lack of experience.

The present research project describes the algorithm to determine the load distribution of planetary gearboxes with double helical gears. The detailed calculation methods offer the possibility to use the actual assembly design for the necessary finite element analysis; results have shown that approximated models cannot be used to calculate the load distribution accurately. Additionally, analytical calculations for the bearing deformation are presented. As already mentioned, the process of combining the single information to a final result is of major importance. The interdependent elements of the assembly lead to nontrivial correlations, making multiple calculations inevitable. The research project also offers a structured approach for the iterative calculation process that allows to be transferred into a computer- aided calculation program. This solution enables the possibility of an effective and detailed calculation process for planetary gearboxes with double helical gears.

Software (Ref.2) has been developed to facilitate the complex calculation of

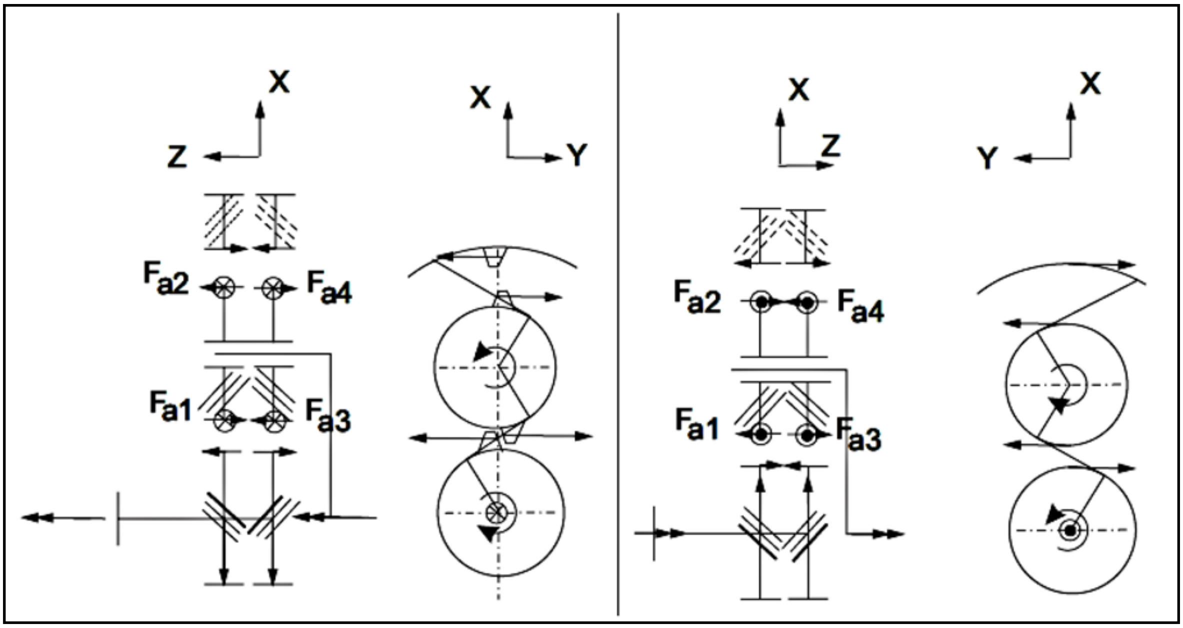

Figure 1 Determination of the loads in double helical-geared gearboxes.

planetary gearboxes and receive information about the whole system with a detailed load distribution along the path of contact of all tooth contacts. Calculations can be made for spur and helical gears in standard, summation and distribution gearboxes. These results are verified by several experiments and analyzed contact patterns of gearboxes in use.

This research project investigates load distribution calculation for double helical gearboxes. Double helical gearboxes are used in different applications in the industry where high rotational speeds, from 500 up to 60,000 rpm, as well as high transmission ratios of up to 35 for two-staged gearboxes, are required. Combined with high torque and transmission of power, the resulting mechanical exposure rises. This study is regarding torque up to 600,000 Nm and power transmission up to 45 MW. Since there are no calculation methods for double helical gearboxes, the research project examines specific characteristics and problems as high torsional deformations, load distribution on the parallel working tooth contacts, and interaction of the left and right gear contact.

Tasks of the research project

Regarding the current calculation options, there are several tasks that have to be added, like extending the contacts, and therefore recalculation of the loads in the gearbox as shown in the example in Fig. 1.

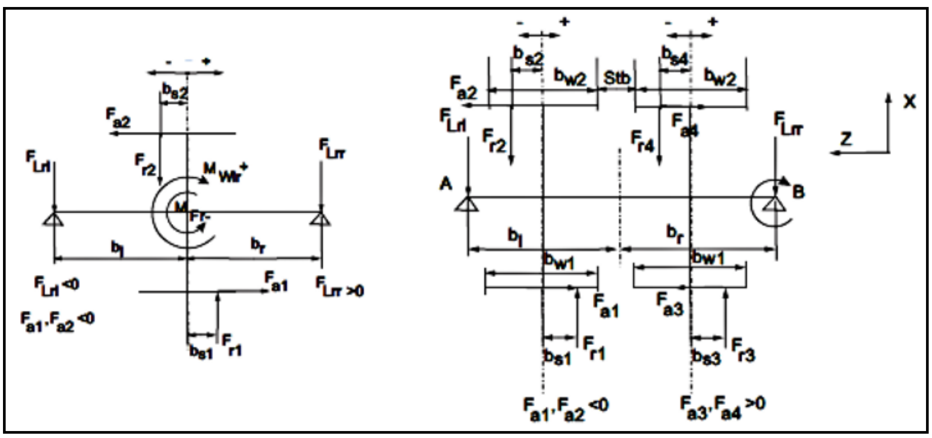

Figure 2 Calculation of planet gear tilt and bearing loads; left: helical gear, right: double helical gear.

Important influence parameters for the load distribution will be affected, like the bearing deformation of the planet bearings caused by unequal bearing loads in consequence of the tilting planet gear with its two tooth contacts. The calculation algorithm of the planet gear tilt and resulting bearing loads has to be extended, as shown in Fig. 2.

The major task for this research is the detailed study of the deformation of the wheel bodies of the gearings. Using approved methods for the load distribution calculation, some specific terms have to be ensured, like that two gear sides of one gear contact shall not influence the deformation of the other gear part more than 1 percent, to exclude errors in terms of the subsequent analytical load distribution calculation.

The realized calculation process shall be used to investigate the behavior of double helical gears that do not occur on helical or spur gears, like a load distribution between the contrary helical gear sides. As there is a default split to 50 percent load on both gear sides, first, results should be developed to intensify the research in the ongoing process of the project.

Need of FEM and analytical calculation methods and their interconnection

The load distribution is affected by various elastic deformations in the system of a planetary gearbox. Generally, the most important influences are caused by tooth deformation, wheel body deformation, shaft deformation, bearing deformation and housing deformation

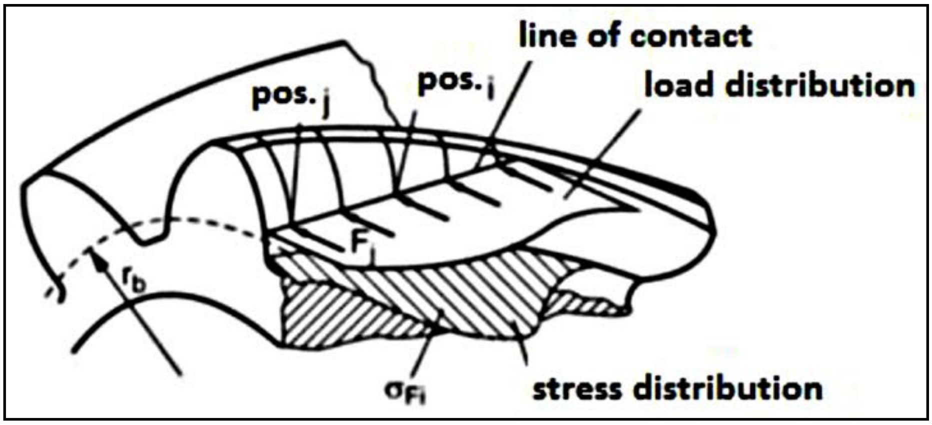

Figure 3 Definition of contact lines and force application points [Ref. 1].

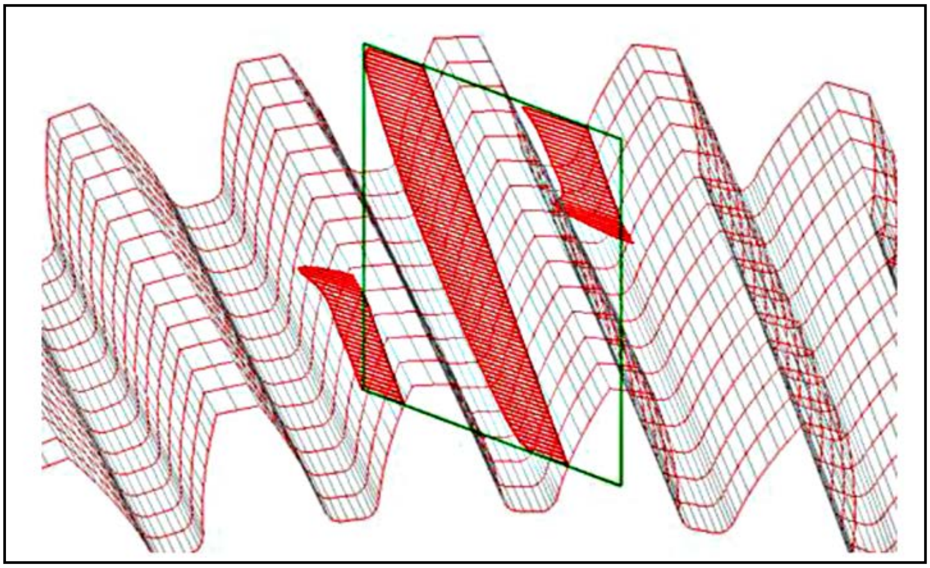

Figure 4 Line load in contact for a single gear side of a double helical gear [Ref. 2].

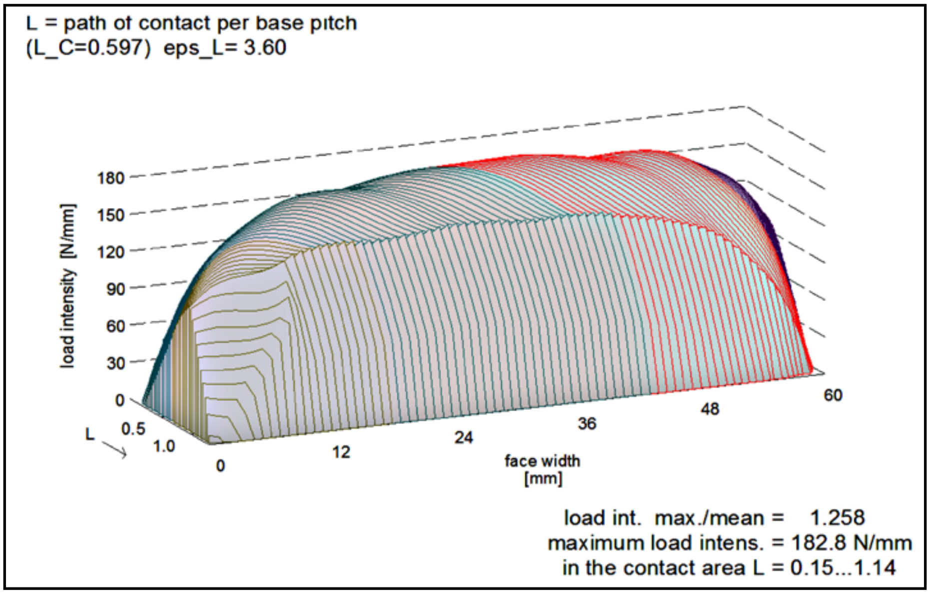

Figure 5 3-D line load distribution in the field of contact [Ref. 2].

The variety and complex interaction of these deformations and their combined influence to the whole system planetary gearbox lead to a time-consuming task to allow a comprehensive statement about the load and deformation behavior. The elements mentioned above can have different causes and consequences, for example: bending deformation, supporting stiffness between tooth and wheel body, shear deformation, Hertzian flattening and compression deformation.

These effects cannot be determined by analytical calculation methods only. FEM models have to be analyzed in terms of deformation and stress concentration under load. The disadvantage of these procedures is high computing times or cost-intensive high-performance computers to achieve efficient development periods. For this reason, there are research projects looking for analytical calculation methods that reflect the results of FEM calculations as precisely as possible. Even though the technical standard of computer-aided calculation rises, finite element analyses and their computing times are not efficient for multiple calculations, as it is needed in a design process or even for further research on specific characteristics in the system. As an example, the calculation software (Ref. 2) uses the influence coefficient method, based on finite element analysis results, to allow real-time calculations of tooth contacts, resulting in a detailed load distribution along the path of contact.

The influence coefficient method is oriented on a bending beam with multiple loads. Every load results in a deformation of all other positions along the beam. The deformations caused by different forces on these positions can be linearly superimposed. Calculating the load distribution, the tooth flanks are separated in several sections for pinion and gear, resulting in a deformation in the contact. Since there cannot be an intrusion or an offset under load, the determined deformation in the contact and the given load lead to the load distribution and transmission error in this contact.

Detailed FE calculations provide the possibility to establish influence coefficients that cover special characteristics

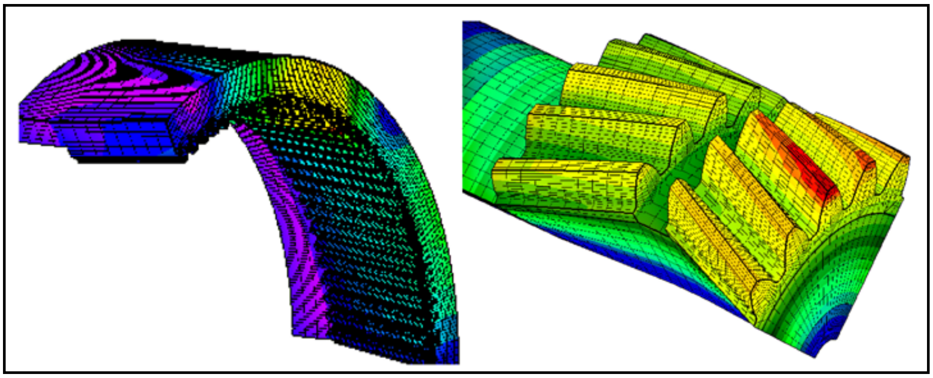

Figure 6 Deformation results for all elements with integrated FE solver [Ref. 2].

of a tooth contact, like multiple teeth in contact, stiffness of the wheel body and stiffness along face width for helical gears, as well as stiffness changes in profile direction or different tooth widths.

Using these influence coefficients and solving the equation system for the deformations in the tooth contact leads to a detailed load distribution. These results have been validated by several experiments, as well as comparisons to running gears and their surface damages.

This approach for the tooth contact analysis can be used in the calculation of planetary gearboxes, but there are no equal calculation methods for all elements. Because of the possibility of any design customized for a specific application, analogous procedures cannot be made for all parts of the planetary gearbox. Therefore, detailed FEM calculations of wheel body, planet carrier and housing deformations are required.

The need for these finite element analyses causes a major complexity in interpreting the system of all elements correctly. All elements in the gearbox interact with each other in terms of load distribution between the elements and therefore deformation changes. The individual result of a single element is an input parameter for the whole system of a planetary gearbox, combining different parts with their specific properties. In order to assure reliable information about a planetary gearbox, an iterative calculation is required.

Automated finite element analysis

The needed FEM calculations have to be as precise as possible to result in system parameters that describe the consequences for the connected parts of this element. Therefore, correct tooth curves are needed to set the loads and contact lines as accurately as possible. The contact angle as well as the contact position in profile and width direction have significant influence on the resulting deformations.

The involute tooth profile described in x and y coordinates is done as follows (Ref. 8):

$$x_y = d_y \cdot \sin(\psi_y)$$
$$y_y = d_y \cdot \cos(\psi_y)$$
$$\psi_y = \frac{\pi + 4 \cdot x \cdot \tan(\alpha_y)}{2 \cdot z} + \mathrm{inv}(\alpha_t) - \mathrm{inv}(\alpha_{yt})$$

The tooth root profile has to be determined iterative under consideration of the manufacturing process, tool, and its addendum coefficient ρ_{a0}. In case of manufacturing with a hob or cutting wheel, the tooth root can be calculated as follows (Ref. 8):

$$x_y = r_{w1} \cdot \sin(\varphi_1) - \left(\frac{\Delta h}{\sin(\psi)} + \rho_{a0}\right) \cdot \cos(\theta)$$
$$y_y = r_{w1} \cdot \cos(\varphi_1) - \left(\frac{\Delta h}{\sin(\psi)} + \rho_{a0}\right) \cdot \sin(\theta)$$

The calculated coordinates have to be transformed from the transverse section to create the tooth along its width. The accuracy of the FE model can be influenced directly, and accurate tooth curves significantly increase the computing time but offer more precise positioning of the force application points of the tooth force. The position of the force application points of a pair of wheels depends on the direction of rotation of the driving wheel and the engaged position. The position of contact, which should be used for the creation of the FE models, is the position in with the shortest total length of the contact lines (Ref. 3).

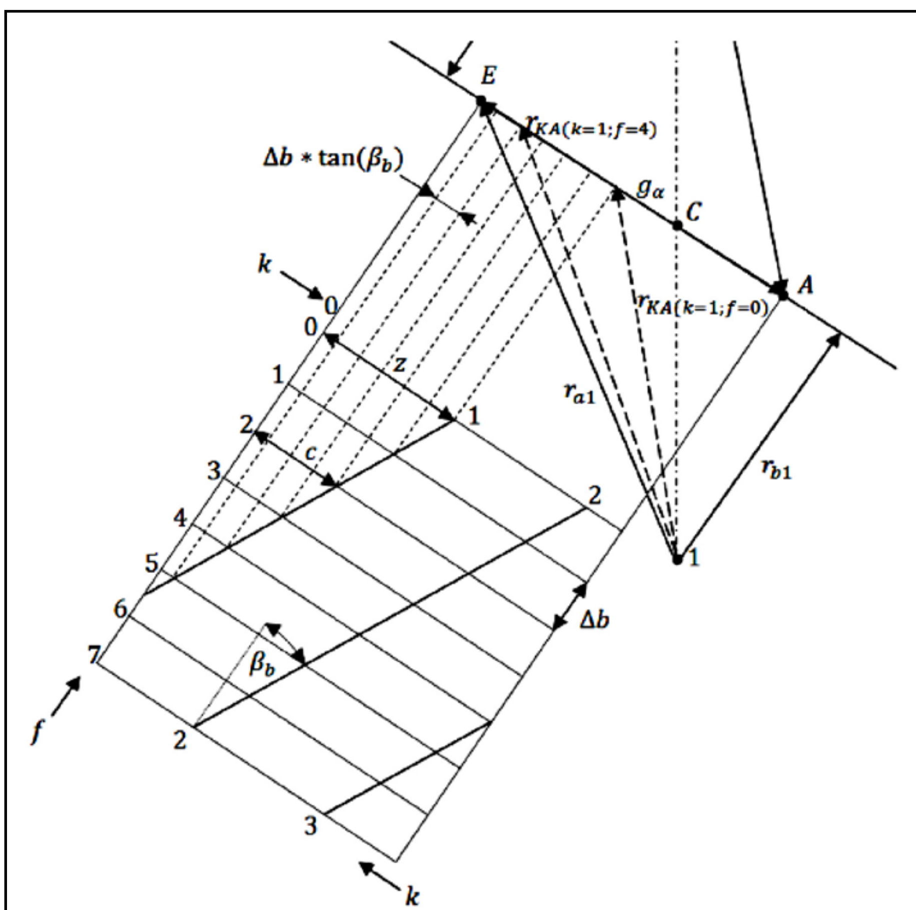

Figure 7 Determination of length of contact lines used in FE models [Ref. 2].

In addition to the contact radii, the correct contact angles have to be determined. Regarding the position of contact, radii of the force application point and contact angle, every single force has to be calculated to result—in addition to the other forces of the application points—in the overall torsional moment of the tooth contact.

The force application points, based on tooth geometry and tooth force, should reflect the real contact line as precisely as possible. Deviations in position and amount of the loads less than 1 percent are considered sufficient.

Furthermore, two load cases — constant and triangular — are calculated for all three gears and the planet carrier. The deformation of a general trapezoidal load distribution can be assembled from the deformation of the base load cases. Because of this, it is not necessary to calculate the FE model for each step of the load distribution iteration.

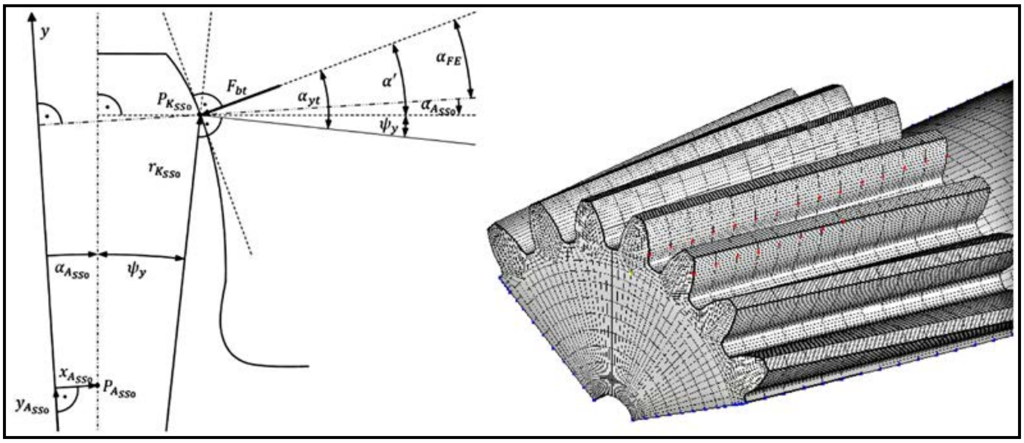

Figure 8 Determination of angles of contact and automated application to FE models [Ref. 2].

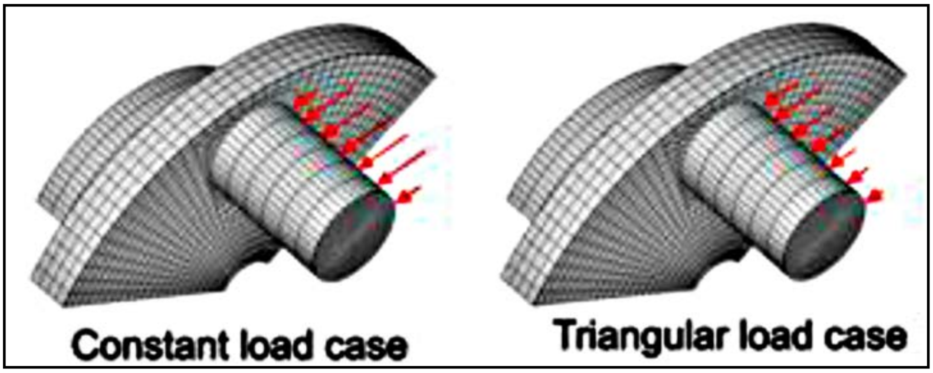

Figure 9 Base load cases for interpolation of load-dependent deformation.

Since there are more tooth contacts in double helical planetary gearboxes, the base load cases as shown in Fig. 9 are extended by additional combinations of load cases between the right and left tooth contact (Fig. 10).

The FEM calculations lead to deformation values of sun gear, planet gear, ring gear, and the planet carrier. All elements have to be interpreted at the points they stay in contact to other elements or have influence to their position in the system. To get to the result of a load distribution, all misalignments and deformations have to be converted into the field of action of the calculated gear pairs and contacts of sun- to-planet gear and planet-to-ring gear.

FEM and analytical methods in an iterative calculation

All results of the finite element analysis and analytical calculations have to be brought together in a calculation

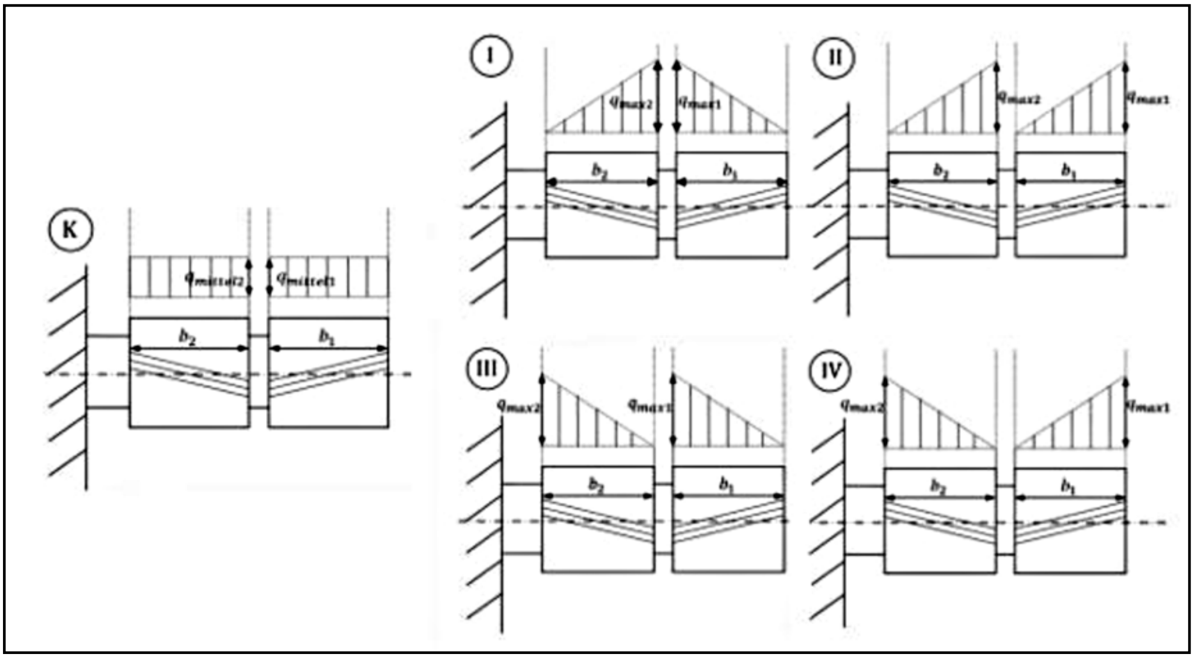

Figure 10 Adapted load cases for double helical gears.

process that covers the interactions between the elements and checks for changes in the system in terms of deformation changes, load distribution changes or even load changes. The calculation routine has to be repeated iteratively until all criteria are changing less than 1 percent. This computer-aided algorithm allows multiple calculations in a short period of time that can be used to improve the design itself or optimize the gearbox with the use of modifications in micro geometry. Especially for double helical gears, there is a need to separate the tooth contacts in terms of modifications since the deformations and the influence of the left and right tooth contact is significantly different to the system. At the point of this research project, there are no possibilities given yet for an automated load distribution calculation for double helical planetary gearboxes. Having this combination of qualified time-efficient analytical load distribution and precise time-optimized, automated finite element analyses offers a new possibility for the optimization in design, space and efficiency of double helical planetary gearboxes.

Optimization

Having the opportunity of multiple automated calculations, the optimization process is more effective. Parametric FE models can be used to analyze the influence of macro geometry and lead to different design goals in terms of weight, size, mass inertia, or stiffness to meet the given requirements. The optimization process should always focus on changes in design and macro geometry. For special agreements or fixed conditions of design space or manufacturing processes, final optimizations can be realized by using modified flank geometry. Especially the characteristics of double helical-geared planetary gearboxes require optimizations in micro geometry in most cases. The difference in deformation amongst the left and right tooth side and the changing helix angle could only be prevented by using very stiff components, resulting in unneeded oversized parts of the gearbox. For this reason, flank modifications in profile and lead direction can be calculated either as preset values for crowning, helix angle modifications, etc., or as a freely defined flank topology.

Summary

The present research project describes the algorithm to determine the load distribution of planetary gearboxes with double helical gears. The detailed calculation methods offer the possibility to use the actual assembly design for the necessary finite element analysis; results have shown that approximated models cannot be used to calculate the load distribution accurately. Additionally, analytical calculations for the bearing deformation are presented and implemented. As mentioned above, the process of combining the single information to a final result is of major importance. The interdependent elements of the assembly lead to nontrivial correlations, making multiple calculations inevitable. The research project also offers a structured approach for the iterative calculation process that allows to be transferred in a computer-aided calculation program. This solution enables the possibility of an effective and detailed calculation process for planetary gearboxes with double helical gears.

In comparison with spur and single helical planetary gearboxes, the advantages and disadvantages are shown in Fig. 12 and Table 1.

As shown in the results above, all gear designs have advantages and disadvantages. The design should be chosen by the goal that has to be realized. In terms of compensating high axial forces, double helical, as well as spur gears, are preferable, whereas double helical gears have better noise emission due to higher contact ratios. Another advan-

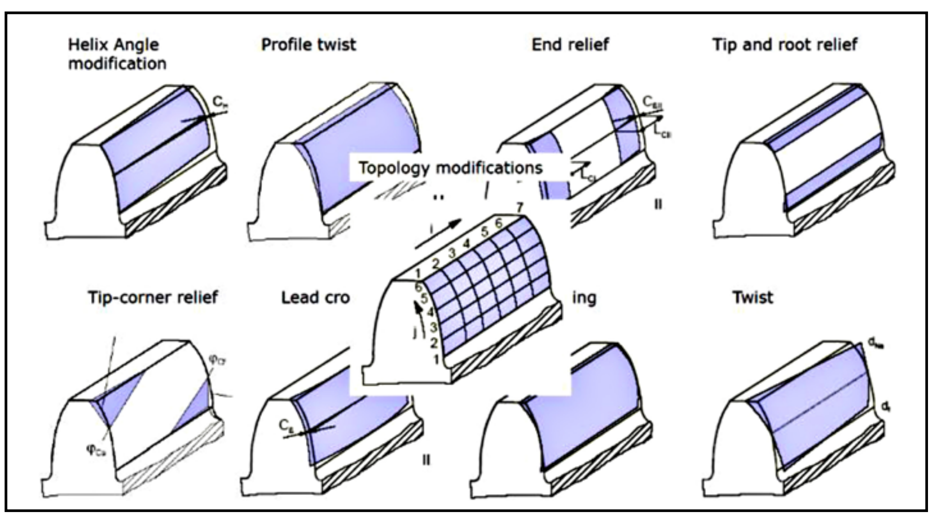

Figure 11 **Modifications in lead and profile direction separately for both sides of double helical gears [Ref. 1].**

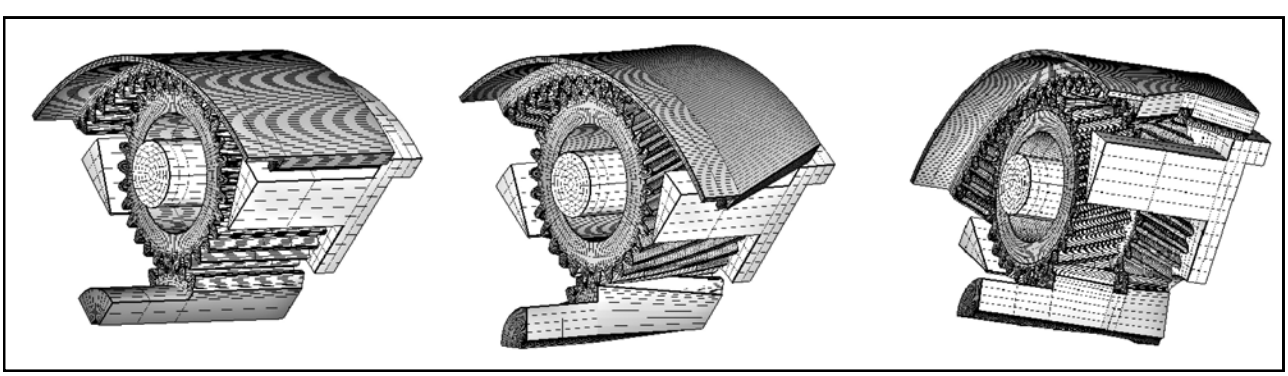
Figure 12 **Comparison of gear designs.**

Table 1 Comparison of gear designs (extract)

	Spur gear	Helical gear	Double helical gears
Power	35 MW		
Speed	2500 rpm		
Max. torque	550000 Nm		
$K_{H\beta}$ unmodified	SU/PL 1,3	SU/PL 1,4	SU/PL 1,1 | 1,5
	PL/RG 1,2	PL/RG 1,5	PL/RG 1,2 | 1,6
Planet bearing deformation — misalignment	70 µm	75 µm	10 µm
Noise emission tooth contact	54 dB	31 dB	35 dB
Axial force	–	86000 N	–
Total contact ratio (effective tooth width equal for all designs)	SU/PL 1,5	SU/PL 3,7	SU/PL 2,6 | 2,6
	PL/RG 1,67	PL/RG 3,8	PL/RG 2,78 | 2,78

tage of double helical gears is the equal load distribution from the planet gear to the bearings. Being almost independent from the actual tooth contact load distribution, double helical gears differ from spur and helical gears, whose bearing deformation highly depend on the tooth contact. A major disadvantage of double helical gears is the expensive production cost, furthermore as shown in Table 1 most of the time there will be a need of different modifications on the gear sides, which may lead to additional costs.

The results given in a detailed load distribution can be used for strength verifications, according to given standards such as ISO, DIN or AGMA. Comparisons and possible further development with AGMA 940, *Double Helical Epicyclic Gear Units*, are follow-up projects after realizing the current research project.

Whereas approximated calculations such as AGMA 927-A01, and the derived ISO 6336 Appendix E, can provide good results in terms of lead modifications and face load factors for standard spur gears, this research project and the developed software (Ref. 2) is able to regard modifications and their consequences in profile and lead directions as well as in the whole field of contact. Specific effects of double helical gears or deformations of the interacting elements in a planetary gearbox cannot be considered in approximated methods. An automated workflow and calculation routine combines different calculation methods such as analytical and numerical calculations and enables this complex calculation to be done by a single operator. Using parametric models and implementing strict check routines for the calculation, interpolation, iteration, and evaluation minimizes error susceptibility for users not having comprehensive expertise of every calculation method used in the project. Optimizing time efficiency, by using calculations with high computing time such as FE calculations only if no analytical algorithm could be investigated, the developed calculation algorithm can be automated and digitalized into a computer-aided calculation process and therefore is qualified for serial calculations and further research projects as well. **PTE**

References

1. Schlecht, B. *Maschinenelemente Band 2*, Pearson Studium, 2015.
2. *MDESIGN LVRplanet* Calculation Software, DriveConcepts GmbH.
3. Baumann, V. “Untersuchungen zur Last- und Spannungsverteilung an bogenverzahnten Kegelrädern, Dissertation TU Dresden, 1991.
4. Hartmann-Gerlach, Christia. “Erstellung eines Berechnungskerns für die Software *MDESIGN LVR planet*.” Unveröffentlichte interne Arbeit, DriveConcepts GmbH 2007.
5. Börner, J. and M. Senf. “Verzahnungsbeanspruchung im Eingriffsfeld – effektiv berechnet,” *Antriebstechnik* 34, 1995, 1.
6. Börner, J. EDV-Programm LVR Programmdokumentation TU Dresden, 1996.
7. Hohrein, A. and M. Senf. “Untersuchungen zur Last- und Spannungsverteilung an schrägverzahnten Stirnrädern.,” Dissertation TU Dresden, 1978.
8. Linke, H., W. Mitschke and M. Senf. “Einfluss der Radkörpergestaltung auf die Tragfähigkeit von Stirnradverzahnungen.,” *Maschinenbautechnik* 32 (1983) 10, S 450–456.
9. AGMA 927-A01. Load Distribution Factors – Analytical Methods for Cylindrical Gears.
10. AGMA 940-A09. Double Helical Epicyclic Gear Units.

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