Phase Management as a Strategy to Reduce Gear Whine in Idler Gear Sets

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Introduction

Gear whine is caused by transmission error, shuttling, friction, impacts, lubricant entrainment and air entrapment (Ref. 1). While the list of gear noise factors is fairly long, it should be recognized that the typical gear noise problem is not a result of lubricant entrainment or air entrapment. Impacts are due to corner contacts that might occur when the teeth just come into contact or due to interference at the root. Both are usually avoided by good gear design. Corner contact is avoided by applying appropriate relief to the teeth. Interference is avoided either by undercutting the flanks of the pinion teeth, by using long addendum pinion and short addendum gear teeth, or by increasing the pressure angle (Ref. 2). That leaves transmission error and other mesh forces.

The mesh forces are usually described as line of action (LOA) and off line of action (OLOA) forces, otherwise known as normal and tangential forces. The greatest line of action forces are from transmission error, which is a line of action displacement, and is the focus of this paper. Shuttling forces of helical gears also act along the line of action. The position of the force vector representing the mesh force shuttles axially as the gears rotate through one tooth mesh sequence. The bearings must react to the axial change in load position and therefore see a change of force that varies at tooth mesh frequency. Tooth friction forces are off line of action forces (Ref. 3). Shuttling and friction forces are generally regarded to be of lesser importance to gear noise but become important when the transmission error is very small.

Transmission error is the driven gear's deviation from perfect conjugate action and is the result of manufacturing geometry errors; gear tooth, shaft and housing deflections; mesh stiffness variation; and dynamics (Refs. 4–6). The dynamic forces generated within the gear mesh because of imperfect conjugate action are reacted at the bearings supporting the gears in the housing. These dynamic forces on the housing excite the housing causing it to vibrate. The housing surface vibration couples with the air causing pressure fluctuations that travel as sound pressure waves to our ears; we hear them as sound. Reducing transmission error is the preferred approach to reducing gear whine.

The forces acting in the gear mesh are made up of DC and AC forces. The DC mesh force is the necessary static force required for the gears to transmit power. This force is present by design. Static forces cause certain design challenges such as bearing life and shaft and housing deflection, but they don't cause gear noise. Gear noise is the result of the AC forces

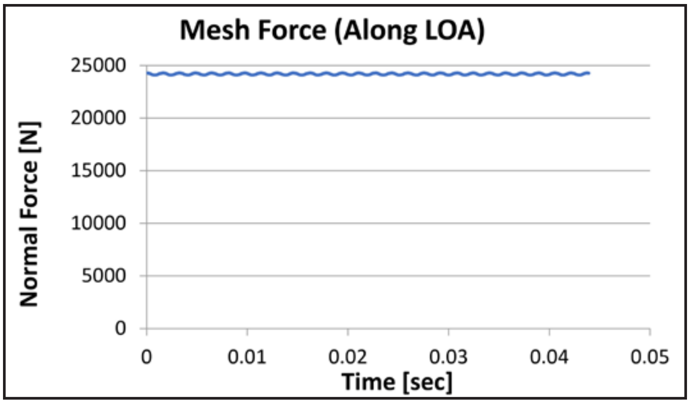

Figure 1 AC forces due to dynamics such as transmission error are very small in comparison to the DC mesh forces.

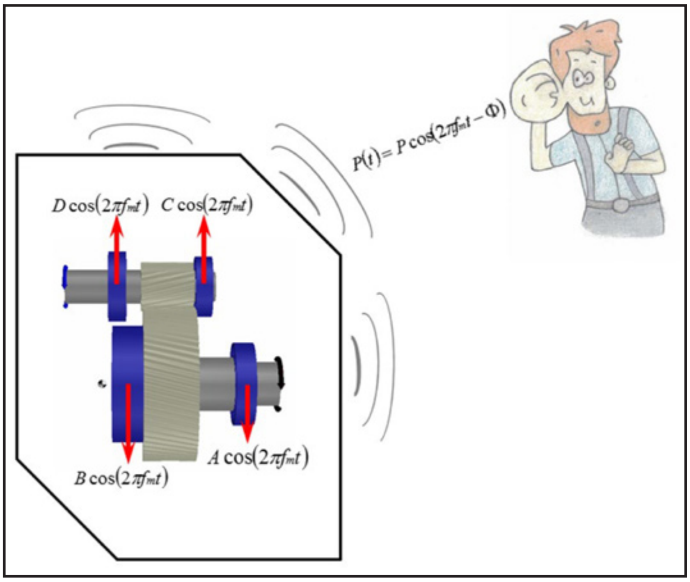

Figure 2 The red arrows indicate bearing forces, from the transmission error forces at the mesh, being applied to the gearbox. Notice all bearing forces for a single mesh are in phase.

from transmission error, shuttling, and friction, and are unintentional. These dynamic forces are tiny in comparison to the static forces. In the example to be discussed later, the DC mesh forces are in the magnitude of the weight of a pickup truck while the AC forces are less than a bag of dog food (Fig. 1). It is only these small AC forces that we are interested in from a noise perspective. Because all the bearing forces are resolved within the gearbox, summing all bearing reaction forces on the housing results in zero, yet there is noise. That is because the forces act on various locations of the gearbox.

Consider the two gear mesh of Figure 2. These transmission error forces at the bearings are in phase, that is, they reach maximum values simultaneously (assuming the gears' and shafts' first natural frequency is sufficiently greater than the mesh frequency).

Transmission error forces are managed by making gear microgeometry modifications. While transmission error can theoretically be reduced to zero at a single load, it is the gear designer's challenge to make transmission error small over a range of loads (Refs. 7–8). Transmissions that operate over a wide range of speed and load are especially challenging and the designer then must make choices about where in the operating space it is imperative to minimize transmission error.

Objectives

The objective of this paper is to show that in idler gear trains there is an additional opportunity for reducing gear whine. The idler forces acting on the idler bearings from the two idler meshes add vectorially to produce a transmission error force ellipse. This ellipse is swept through once per tooth mesh. It will be shown how one may manage the phase between the two meshes on the idlers to minimize the size of the ellipse. If the sum of the transmission error forces is reduced, the reaction forces at the bearings are reduced. Generally, reducing dynamic forces on the housing reduces noise.

The size and orientation of the transmission error force ellipse is managed by three "knobs," they are the number of teeth on the idler, idler tooth thickness and idler position (working pressure angles and which side of the line of centers connecting the input and output gears—things that affect the orientations of the lines of action). Rest assured, one still must well manage transmission error because even if the forces at the idler could be reduced to zero, there remain transmission error forces acting on the input and output gear bearings.

Designers are provided insights for reducing gear noise in idler gear systems. These concepts are demonstrated by means of a case history of a PTO gearbox whereby making phasing changes led to reduced gear whine.

Phase Study in Previous Work

The importance of phasing gears has been long appreciated in planetary circles. Schlegel and Mard claim noise reductions as much as 11 dB by using "system phasing," a strategy that sequences the meshes between pinions to "eliminate any external force or moment reaction" (Ref. 9). Independently, Seager developed a concept for "neutralizing" harmonic components of tooth excitations by carefully choosing the number of teeth on the sun and ring gears (Ref. 10). Neutralizing applies only to planetary gearing with equally spaced planets and complete neutralization can be obtained for torsional modes or transverse modes, but not both. Palmer and Fuehrer provide a table which indicates the neutralized torsional and transverse modes for various numbers of teeth on the sun gear (Ref. 11). They then introduce "counter-phasing" as the next step in the progression of planetary gear phasing. With counter-phasing, the planetary torsional stiffness variation is reduced, assuming that torsional modes are more important than transverse modes. Torsional modes are optimally neutralized "when the pinion phase angles φ_i are equal to different multiples of 360°/n, where n is the number of pinions."

Kahraman and Blankenship extend the planet phasing ideas from a kinematic study into the dynamic realm (Ref. 12). Transmission error functions at the sun-planet and plant-ring meshes provide the dynamic excitation. The authors concluded that no one phasing option was indisputably better than another and that the best phasing choice may be application dependent. Similarly, Parker examined planet phasing for unequal planet spacing (Ref. 13). He shows how one can predict when rotational and translational or both forces cancel, thereby not causing excitation even while traversing a resonance.

Regarding parallel shaft gearing, Muehl and Sternfeld built a test rig with two gear meshes; this gearbox had two gears with the same number of teeth on the input shaft (Ref. 14). Load was applied to the two output shafts. A coupling between the two input gears allowed them to adjust the relative phase between the meshes. While they concluded that changing phase affected the measured sound power, they could not make a recommendation for what relative phase would result in the least noise as their results were confounded with other parameter changes such as different teeth in mesh.

Kubur, et al. focused on dynamic analyses of countershaft gearboxes (Ref. 15). One of the items they investigated was how bearing forces change with changes in the shaft layout as viewed along the shafts' axes. In their analyses the idler was a compound gear with different tooth counts for meshes with the input and output gears.

Cheon suggests one can reduce mesh stiffness variation in parallel axis spur gearing by adding a second set of identical gears phase shifted by half a tooth (Ref. 16). This is somewhat akin to double helical gears which is tantamount to two gears side by side but with opposite hand.

Brecher, et al. studied a two-stage gearbox (Ref. 17). Some of their runs were with both gears on the intermediate shaft with the same number of teeth. These were run in phase and shifted half a tooth. By shifting one-half tooth, their difference velocity level measurements reduced 3.5 dB for the fundamental mesh frequency and reduced 2.5 dB at the third harmonic (Difference velocity level is the measured transmission error in terms of angular velocity converted to dB with reference velocity 1e-6 m/s. The authors measured angular acceleration with two tangentially mounted accelerometers and integrated the signal to angular velocity.) There was no change at the second harmonic.

Kartik and Houser exercised a model to perform parametric analyses on an idler gearbox (Ref. 18). The computed dynamic force responses at each of the six bearings were used to predict the housing surface accelerations that were then used to calculate sound power. They investigated the effects of bearing stiffness, mesh stiffness, housing wall stiffness and mass, etc. They then ran simulations manipulating the phase of the two idler meshes. The baseline condition was the original gearbox arrangement, nearly in phase. They also considered the two meshes perfectly in phase and 180° out of phase. They found that the idler shaft bearings responded to the phase change, as expected, but the input shaft bearings responded as well. They reported "a significant drop in the sound level at the lower frequencies when the meshes are in opposite phase, with a maximum reduction of about 45–50 dB at the peak of lowest natural frequency."

Liu and Parker compared various techniques for dynamic modeling of non-linear idler gear systems (Ref. 19). They compare lumped parameter models, the harmonic balance method and perturbation analysis. These were then compared against the numerical integration method.

The Transmission Error Force Ellipse

The importance of phase will be underscored by considering virtual gear noise analyses. In a gearbox with an idler like the schematic shown in Figure 3, there was an inordinate amount of gear noise. The input and output gears were supported by the stiff bearing supports on the ends of the gearbox, however the idler support was directly tied to the surface of the housing. While the idler bearing stiffness was sufficient to carry the static bearing loads, it is thought that applying transmission error forces directly to the housing skin is an efficient way to cause the housing surface to vibrate and therefore transmit noise. This idea was reinforced by other gear pairs in the same gearbox with similar calculated transmission error that went directly from the input to output shaft, not employing an idler, and not causing gear noise problems.

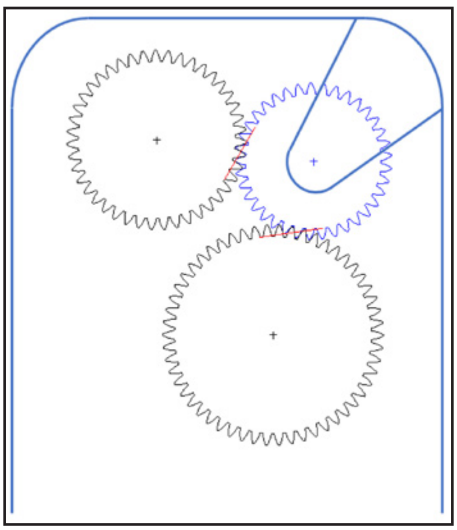

Figure 3 Gearbox with an idler set—input at the top (CCW rotation) and output at the bottom.

To further explore this theory, the transmission error reaction forces were applied to the housing at the bearings for the three gears, all together and individually. The response variable for this analysis was the gearbox average surface velocity since noise is generally assumed to be related to surface velocity. Figure 4 shows that the response due to the transmission error forces acting on the input (DriveR) and output (DriveN) shafts alone, shown in blue and green, had minimal contribution to the total response. The response from the idler transmission error forces, shown in red, however, was nearly equal to the total response, shown as light blue.

In this phase analysis, transmission error forces were assumed to be at the fundamental mesh frequency and have unit amplitude. Phase was calculated by the method outlined in Appendix A. The phase is a function of the number of teeth on the idler, idler tooth thickness and the directions of the lines of action. The transmission error forces from the two meshes of the idler are summed by vector addition at every instant in time. These are only the tiny AC forces that ride on top of the very large DC forces; that is why they appear as sinusoids. As the gears rotate through a single tooth mesh cycle, the summation of the force vectors at the idler sweeps out an ellipse (Fig. 5).

The size, shape and orientation of the ellipse can be managed, however, by carefully selecting idler parameters such as number of teeth, tooth thickness and its position. Since

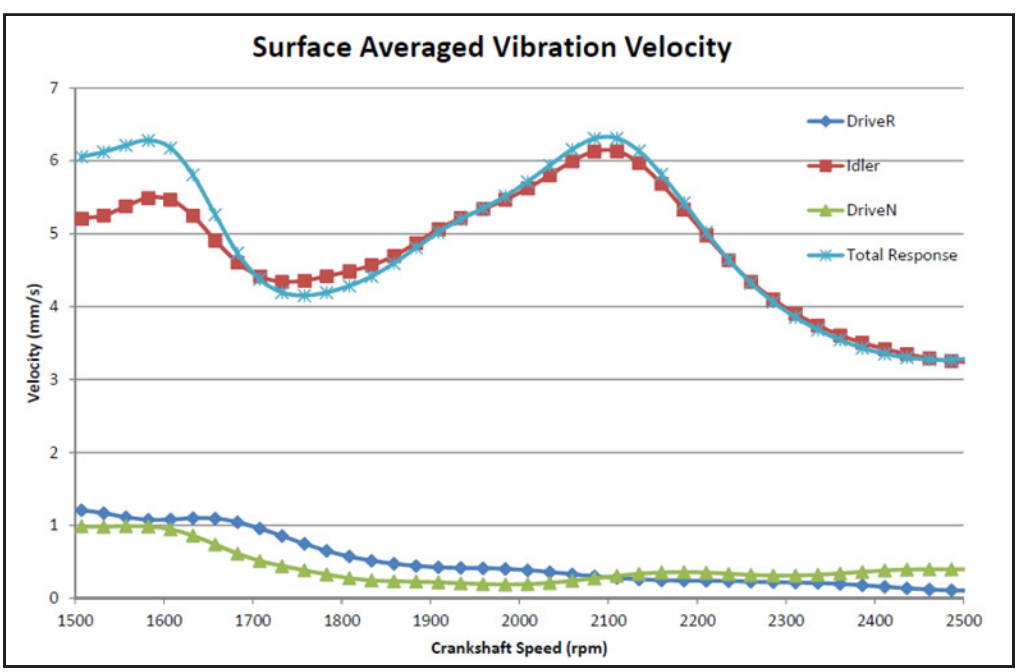

Figure 4 Average surface velocity on the transmission case with bearing forces applied to the housing of Figure 3 one shaft at a time, and all together, for a total response.

the force vectors are parallel to the lines of action, the principal axes of the ellipse bisect the two idler lines of action. This means that the orientation of the ellipse is defined by the location of the idler. It is not yet determined which is the major and which is the minor axis; that requires the number of teeth and tooth thickness.

To demonstrate the sensitivity of the ellipse to the number of teeth, the transmission error force ellipses are calculated for many different idlers with the same tooth thickness (Fig. 6). Because the number of teeth changed, the idler center had to change accordingly — these have constant pressure angles. Because the pressure angles were kept constant, the alignment of the ellipse axes is maintained.

We can see that in this over-plot of many ellipses, some are much smaller and others are rotated 90 deg. Let's look at a few of these ellipses more closely. Notice in Figure 3 that the idler is connected to the housing at the upper right. It is therefore logical that forces in that direction are very efficient at adding vibration energy to the housing skin. Likewise, ellipses with major axes oriented up and to the right would produce more surface vibration. The black ellipse of Figure 7 is the baseline idler ellipse of the gear train analyzed in Figures 3 to 5. The 18T ellipse (blue) might be a better choice because it is significantly smaller than the black ellipse. The 31T ellipse (red) is even shorter in the direction of the 37T major axis and perhaps would produce less noise. One would expect, however, that it would produce more side to side motion, but this might not produce as much radiated sound. Depending on how the idlers bearings are attached to the housing, the ellipse orientation may be important, so an acceptable choice need not be a small ellipse, just be in a favorable orientation.

The effect of changing tooth thickness without changing the number of teeth or the position of the idler has the effect

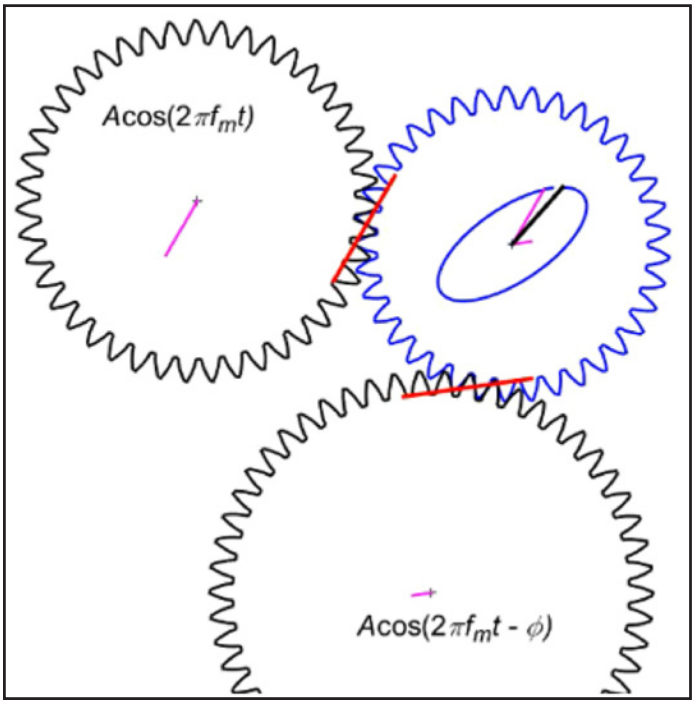

Figure 5 Unit amplitude transmission error force vectors (pink), parallel to the lines of action (red), are applied to the gears and reacted at the bearings. The forces are sinusoidal at mesh frequency and phase shifted. At the idler, the summation of the transmission error force vectors from the two meshes are added to make the resultant vector (black). The resultant vector sweeps through an ellipse as the gears rotate through a single gear tooth mesh cycle.

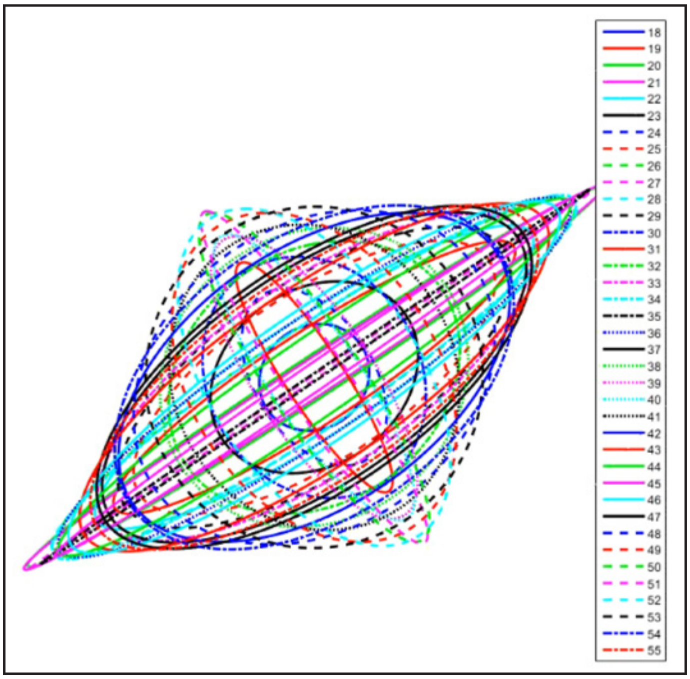

Figure 6 Collection of transmission error force ellipses generated by changing idler tooth count from 18 to 55 teeth while maintaining the same tooth thickness and the same pressure angles.

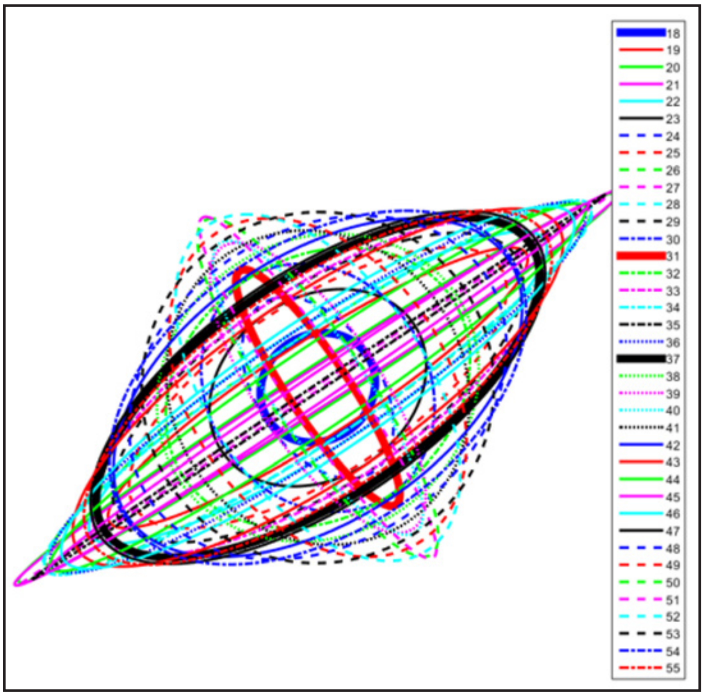

Figure 7 The transmission error force ellipses of Figure 6, highlighting specific ellipses.

shown in Figure 8. While the ellipse can be affected, it cannot be manipulated nearly as much as by changing the tooth count, mostly because tooth thickness can only be changed so much.

Transmission error forces act along the lines of action, which in turn, define the orientation of the ellipse. That

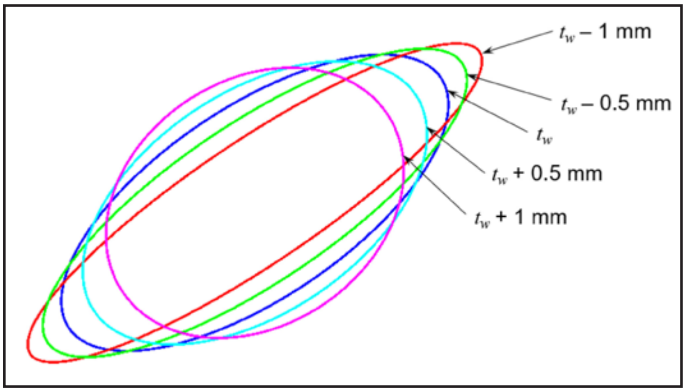

Figure 8 **Transmission error force ellipses from the gearbox of Figure 5 showing how their sizes and shapes are affected by only changing idler tooth thickness; baseline tooth thickness is t_w.**

means there is significance to which side of the line of centers connecting the input and output gears that the idler is on. On one side, the lines of action tend to be more parallel and on the other more perpendicular. One can't produce a small ellipse if the lines of action are perpendicular. On the other hand, when the lines of action are parallel, the DC bearing forces are greater and bearing life is reduced. Such are the compromises of the designer!

Manipulating the phase at the idler likewise affects the phase at the output gear relative to the input gear. While controlling the input and output phases on the input and output shaft bearings is a theoretical method for reducing the sound radiated off the housing, it is not very practical, because if it works, it is only over narrow speed ranges. Furthermore, identifying the desired relative phase that would minimize gear noise is not trivial.

Describing the Size of an Ellipse

Since we expect that the size of an ellipse is important in determining which idler design is better than another, it is important that we have a way to describe an ellipse's size. In some cases, only the dimension in a specific direction may be important, i.e. direction cosines, describing the angle off the horizontal to the major axis. In the general case we want to capture the overall size of the ellipse regardless of orientation. While area of the ellipse seems to be a natural choice, there are ellipses that are flat (the minor axis is zero) therefore the area is zero. This is much different than an ellipse that collapses to a point because the lines of action are parallel and the phase is exactly 180°.

The size descriptor proposed is the root of the sum of the squares of the major and minor axis lengths as shown in Figure 9 and Eq. 1. Because we are addressing noise, it is desirable to also have a dB descriptor; one is proposed in Eq. 2. Notice that its format is similar to definitions for vibration or sound level in dB, making it suitable for comparing one ellipse against a reference ellipse, perhaps the baseline design, when one exists. Alternatively, RSS_0 could be unity.

$$RSS = \sqrt{a^2+b^2} \tag{1}$$

$$\mathrm{dB} = 20\log\left(\frac{RSS}{RSS_0}\right) \tag{2}$$

Underlying Assumptions

The two underlying assumptions for this phasing work are that the magnitudes of transmission error forces are approxi-

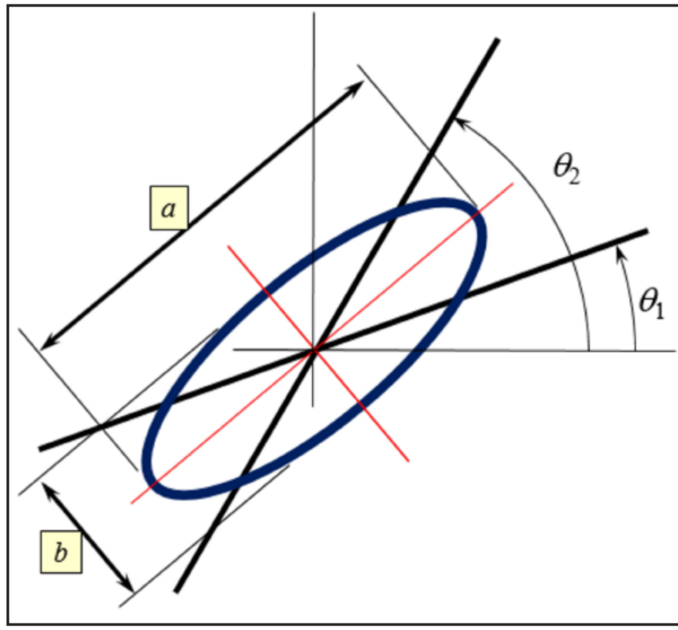

Figure 9 **Descriptors for comparing size of ellipses; the *a*-dimension is halfway between the lines of action, oriented at an angle of $(\theta_1+\theta_2)/2$.**

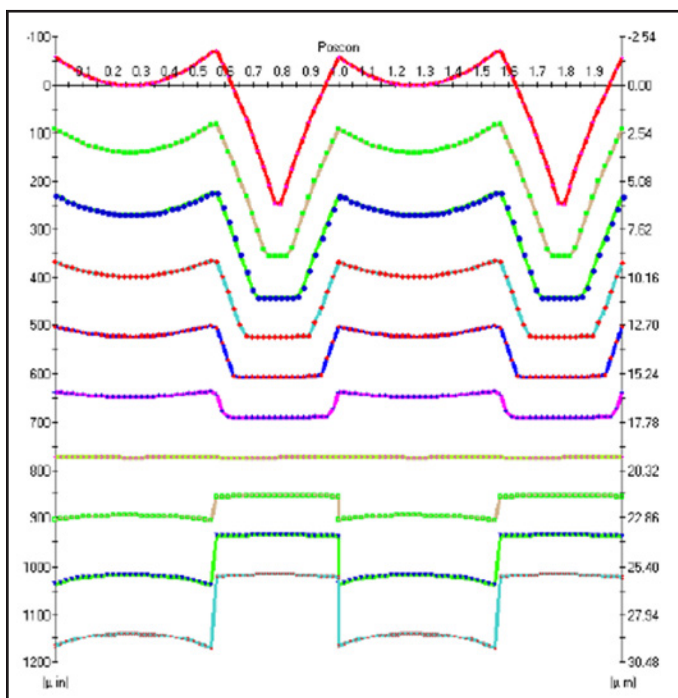

Figure 10 **Harris map of low contact ratio spur gears at various loads with profile modification optimized for a specific load; the discontinuities are the highest and lowest points of single tooth contact (from Ref. 1).**

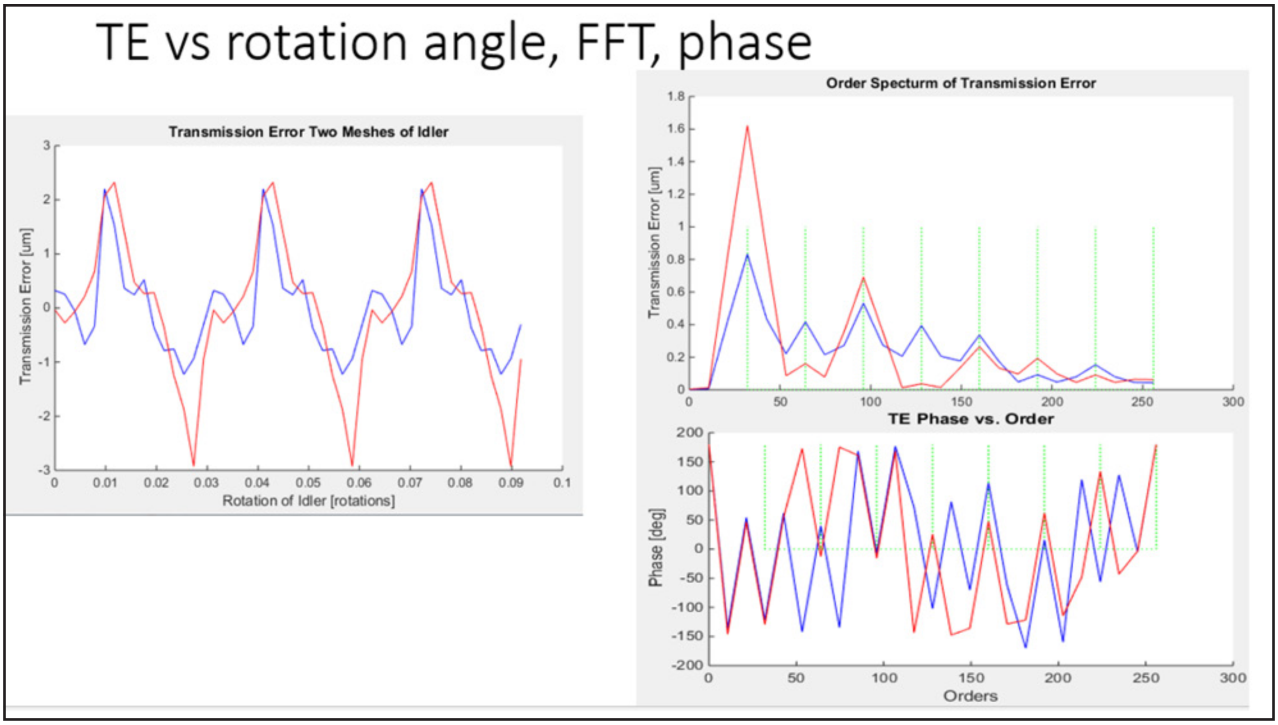

Figure 11 Transmission error vs rotation for the two meshes of an idler and the corresponding FFT with amplitudes and phase. Vertical green lines in the frequency plots indicate harmonics of mesh frequency.

mately equal at the two idler meshes and the transmission error; error forces are symmetric about the midpoint between the lowest and highest points of single tooth contact. These assumptions are now addressed.

The TE force ellipse up to this point was generated by using unit force vectors parallel to the lines of action acting on the idler, with the proper phase. What if the TE forces are not equal? How does one know the relative phase of the transmission error forces between the two meshes? Transmission error forces are not the only gear mesh forces acting on the idler bearings. Must we account for those other forces? Only the fundamental mesh frequency has been considered. What about harmonics?

One might expect the transmission error forces to be of roughly the same magnitude on each mesh of the idler since the manufacturing methods and design philosophies are likely the same. One can further expect that the forces will tend to be symmetric about the midpoint between the lowest and highest points of single tooth contact by inspecting Harris plots (Fig. 10). Figure 11 shows that the transmission error waveforms are somewhat similar for the two meshes and fairly aligned relative to the midpoint between the lowest and highest points of single tooth contact. This plot was created by using the transmission error vs. roll angle data from a virtual analysis, but then plotting it as TE vs. idler rotation.

The FFT (Fig. 11, upper right) shows that for one mesh (red) the amplitude of the transmission error at the fundamental frequency was significantly greater than the other mesh (blue). How does that stack up against the assumption that transmission error forces are the same magnitude? Figure 12 plots, in blue, the sizes of the ellipses of Figure 6, normalized by dividing each RSS magnitude by the maximum RSS magnitude. In this first analysis, both TE force vectors were unity.

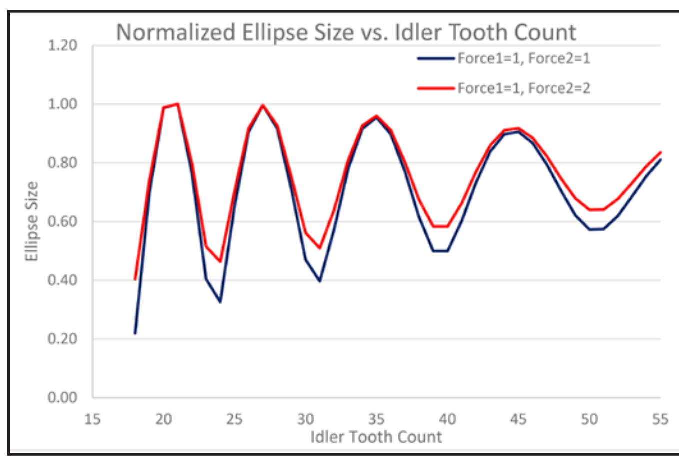

Figure 12 Normalized RSS for ellipses of Figure 6 by first considering both TE forces as unit forces (blue) and then with the second force twice the magnitude of the first (red). Normalized RSS = RSS/max(RSS).

Then the procedure was repeated with the second transmission error force vector of magnitude twice unity and again normalized by dividing each RSS magnitude by the maximum RSS magnitude. While the sizes of the ellipses in the second analysis are greater in size than the first ones, their normalized sizes, in red, show the exact same trends. Even if the transmission error force magnitudes are unknown, this strategy is still useful for selecting idler gear parameters such as number of teeth and tooth thickness.

It is straightforward to see how the sum of the fundamental frequency force vectors may be minimized, but what about the higher harmonics? By reducing the transmission error force ellipse in the manner discussed, the odd harmonics will be minimized, but the even harmonics will be maximized. In the same way, the problem can be reformulated to minimize

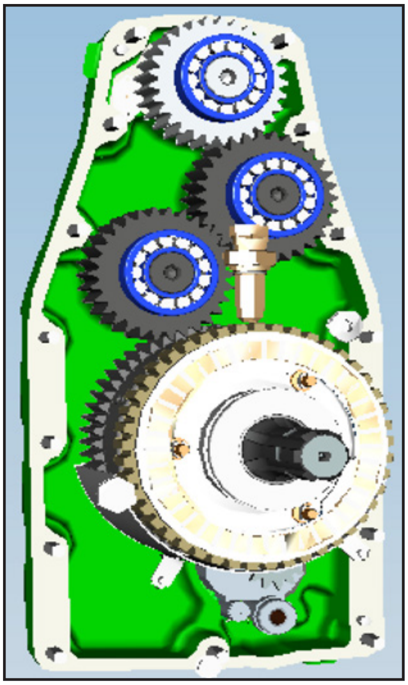

Figure 13 Front PTO gearbox with two idlers; the input is at the top (CCW rotation), then gears 2 and 3 and the output at the bottom.

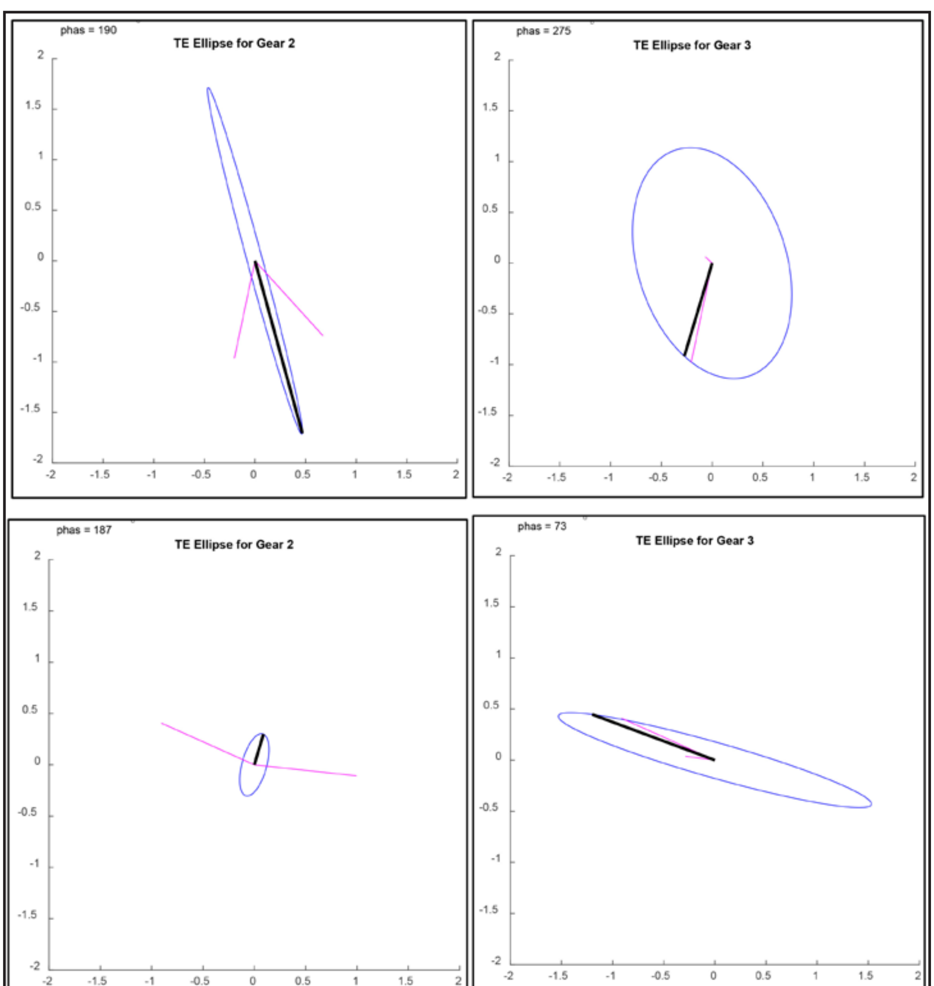

Figure 14 Front PTO transmission error force ellipses for the two idlers for forward (above) and reverse (below) input speed directions.

the even harmonic force ellipse for a challenging gear noise that is primarily second harmonic in nature. Because our gear tooth modifications and microgeometry tolerancing are mostly for the benefit of the fundamental harmonic, this is best controlled. Higher harmonics are of less predictable control. Why the even harmonics are maximized when the odd harmonics are minimized is shown in Appendix B.

Front PTO Gearbox

Early in the development cycle a front PTO gearbox suffered from gear whine. Power is transmitted through three meshes using two idlers (Fig. 13). This gave the authors an opportunity to apply the phasing concept. A quick analysis indicated that the TE force ellipses might be reduced by running the gears backwards by reversing the input shaft direction (Fig. 14), not as a solution, but to give credibility to the phasing concept and to get buy-in from design. Phasing analysis suggested that the total RSS for the reverse input direction would be about 3 dB less than the forward direction. The gearbox was tested by simply reversing the input direction and the sound pressure level indeed reduced about 3 dB. We now had their attention!

The maximum noise contribution was from the first harmonic, so the objective was to minimize the first harmonic by means of phasing, but keeping within certain constraints put forth before us by design. A *Matlab* program evaluated thousands of design permutations of idlers with different numbers of teeth, tooth thicknesses and positions. The proposed design reduced the first harmonic by 2 dB (Fig. 15). When tested, the overall noise was reduced 3 dB, but the second harmonic was now a problem due to a structural resonance (Fig. 16). Virtual and physical modal analyses determined that the second harmonic of mesh frequency excited the ninth mode of the housing. Stiffening the housing put this mode outside the operating range, providing a design that was 6 dB quieter (Fig. 17).

Caveats and Warnings

Earlier it was stated that the position of the idler affects the transmission error force ellipse and that the ellipse is aligned with the bisectors of the lines of action (assuming the transmission error forces are equal). While the size of the ellipse can be modified by changing the number of idler teeth and idler tooth thickness, the ellipse can only be reduced so much if the lines of action are perpendicular rather than parallel. In the best case, the lines of action are perfectly parallel, the forces are equal and phased 180° and the odd harmonics are perfectly cancelled. But if the lines of action are perpendicular, no amount of phasing can reduce the ellipse size smaller than a circle of unit radius. The total potential of a design with perpendicular lines of action therefore is only 3 dB. Whether the lines of action tend to be more parallel or perpendicular depends on which side of the line of centers connecting the input and output gears that the idler is on, as indicated in Figure 18.

When selecting which side of the line of centers of the input and output gears to place the idler, the designer must consider the DC bearing forces on the idler. While

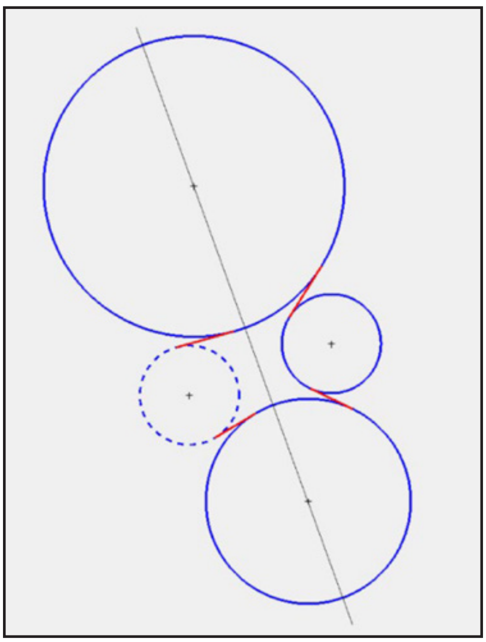

Figure 18 **The lines of action for the idler on the right are roughly perpendicular and therefore provide limited opportunity for reducing the TE force ellipse; the left idler — shown as broken lines, however — has nearly parallel lines of action, providing the best opportunity for minimizing the size of the TE force ellipse.**

there is potential to have the AC transmission error forces cancel by having parallel lines of action, this design results in the greatest DC bearing forces. And since bearing damage is proportional to force to the power of 3 or 3.33, the difference in using a parallel or perpendicular design can potentially affect bearing life by a factor of 3.

Up until this point, we have focused on the fundamental gear mesh frequency. The ellipse concept works for any harmonic of mesh frequency. The caveat is that even and odd harmonics cannot be perfectly canceled simultaneously. When

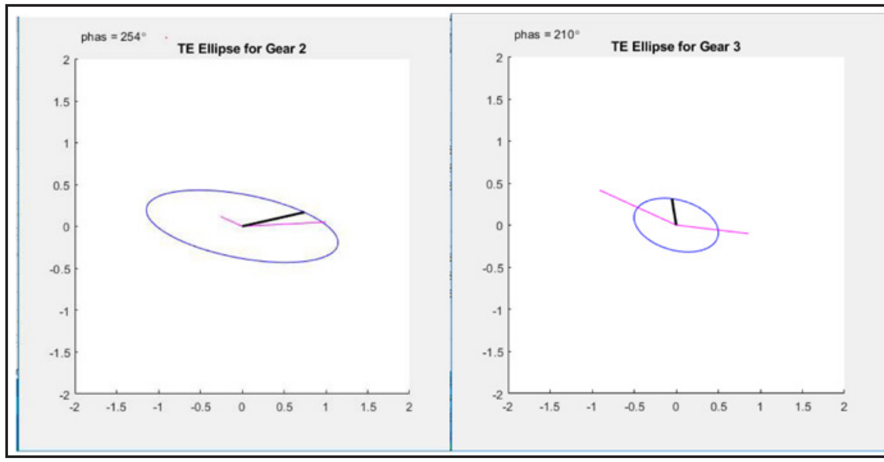

Figure 15 **Front PTO transmission error force ellipses for the two idlers of the new design.**

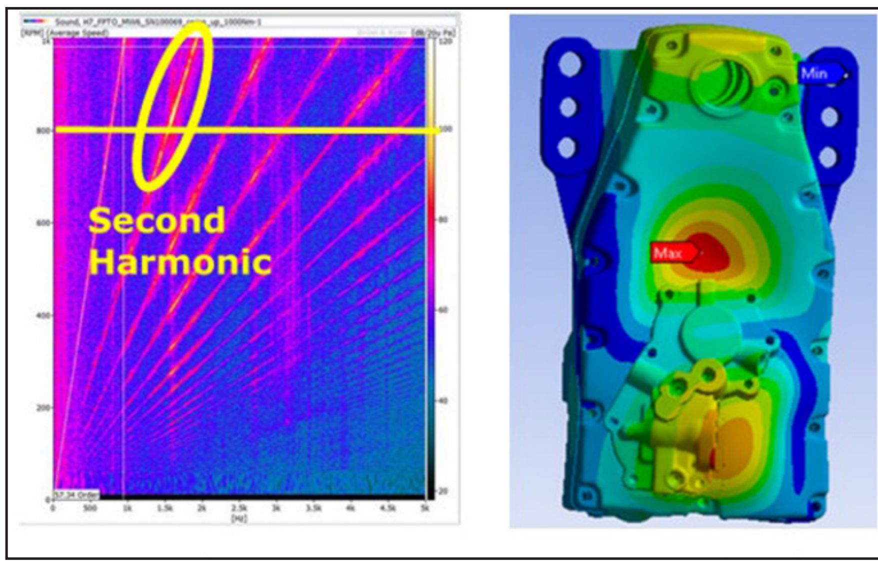

Figure 16 **The waterfall plot highlights the problematic second harmonic of mesh frequency; the cause is the 9th mode shape shown at the right.**

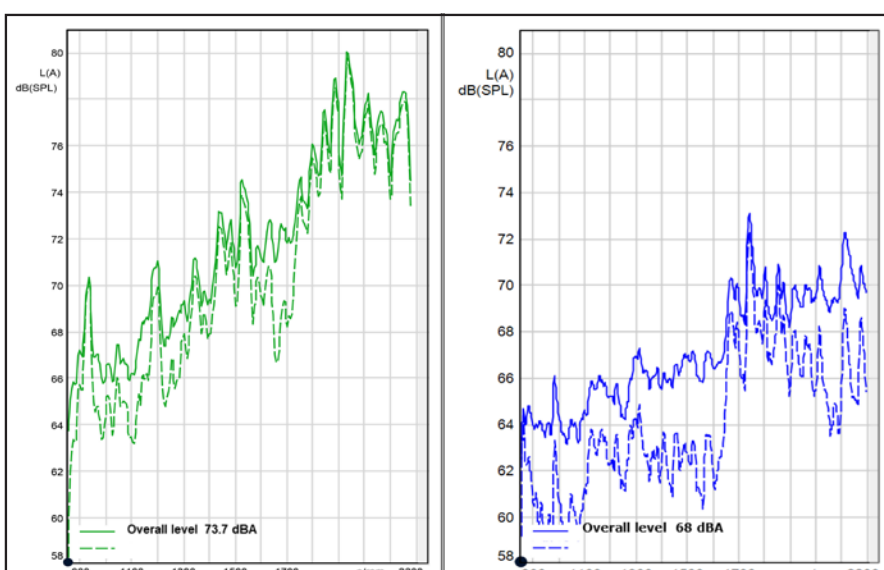

Figure 17 **Overall and sum of the first four harmonics of sound pressure level for the original design on the left, and the final design on the right.**

the odd harmonics are minimized, the even harmonics are maximized. Therefore, one must decide which harmonic is most important to minimize or do a weighted optimization. Because off-road gear tolerances do not well control the higher harmonics of transmission error one might opt to focus on the fundamental mesh frequency. This odd and even harmonic phenomenon is further described in Appendix B. For example, where the 18T idler of Figure 7 was one of the smallest ellipses, for the second harmonic, the 18T ellipse was one of the longest ellipses. The smallest ellipses for even harmonics are obtained when the phase shifts are about ¼ tooth (90°) (Fig. 19). This is consistent with the results described by Cheon and Brecher, et al. (Refs. 16–17).

Quick Method for Verifying Whether Phase May Have Merit

One desires to reduce gear noise but does not wish to send the design team on a wild goose chase to redesign and procure parts to explore this theory. A simple way to determine whether this approach may be fruitful for a given application is to run the input in the opposite rotation direction. Quite often the transmission error force ellipses are quite different, and it gives one an opportunity to predict a change and then observe it. Not all gearboxes can be run backwards, but at John Deere the authors have made provision to run several gearboxes with reversed input shaft direction just for this purpose.

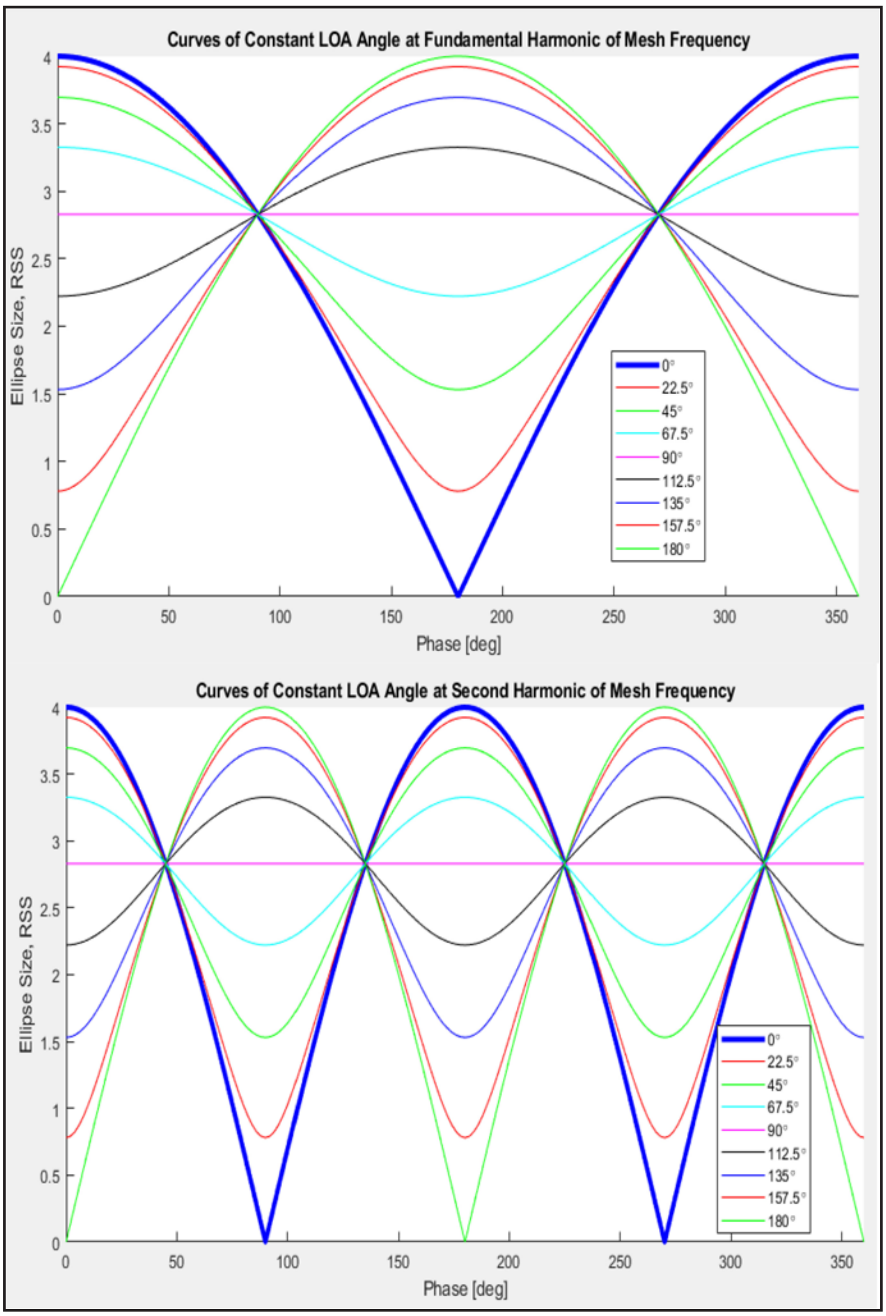

Figure 19 The size of ellipses generated for phase increasing from 0 to 360° with lines of action at various relative angles ($\theta_2 - \theta_1$) increasing from 0 to 180°. The upper plot is an analysis of the fundamental mesh frequency, and the lower plot is for the second harmonic; the bold blue curves denote lines of action that are parallel.

Summary

This work explains why some idler sets produce so much gear whine. While transmission error must be managed, as it is generally the greatest source of gear whine, there is another tool in the gear whine management toolbox the designer can make use of, phasing the meshes on the input and output sides of the idler. The TE forces on the two lines of action acting on the idler add together to make a resultant vector that sweeps out an ellipse as the gears are advanced one tooth. By adjusting the relative phase of these meshes, the transmission error force ellipse size and orientation can be manipulated. The three parameters that affect the ellipse are the number of teeth on the idler, idler tooth thickness and the orientations of the lines of action. The TE force ellipse is minimized when the phase is 180° for odd harmonics and 90° for even harmonics.

A case history of a PTO gearbox was provided that demonstrated how the gear whine was improved by making phase changes.

Design by means of phasing idlers offers no guarantees, but it does offer some possibilities. When tackling a tough gear noise problem for a specific mesh frequency harmonic, it is good to be aware that there is another design option to investigate.

The contents of this paper are protected by U.S. Patent 10,423,756 (Ref. 20).

For more information. Bob White consults as an NVH and Dynamics Engineer with White Noise, LLC and Pravin Pail is a Drivetrain NVH Engineer at John Deere. They may be contacted at: RobertWhite@WhiteNoiseUSA.com and patilpravin2@johndeere.com. They welcome your comments.

Recommendations for Future Work

The current work focused on reducing the transmission error force ellipse size for a single family of harmonics (odd or even). Ideally all harmonics with their individual amplitudes and phases would be considered. This is easy enough to accommodate once a design is completed, but how does one guide designers early in the design? Perhaps many gear designs might be studied to determine typical relative amplitudes and phases and then apply them as rules of thumb.

Future work could also add the off line of action friction forces to the line of action transmission error force vectors, thereby generating force profiles that represent all the idler forces on the housing.

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Appendix A

This appendix develops the equation that computes the relative phase angle between the two meshes of an idler and the dimensions of the TE force ellipse. The phase equations are derived in the format of (Ref. 21) by evaluating angles around the idler from one mesh to the other. Parker and Lin use a different approach; calculating the phase of a planet gear in an epicyclic system by computing the linear distance from one pitch point to the other along the lines of action and around the base circle of the planet (Ref. 22).

Consider Figure A.1, which shows the mesh that drives the idler. Assume these gears are at the point half way between lowest and highest points of single tooth contact. The roll angles at those points are:

$$\lambda_{HPSTC} = \frac{C_1 \sin \eta_1 - \sqrt{r_{A1}^2 - r_1^2}}{r_2} + \frac{2\pi}{z_2} \tag{A.1}$$

$$\lambda_{LPSTC} = \frac{\sqrt{r_{A2}^2 - r_2^2}}{r_2} - \frac{2\pi}{z_2} \tag{A.2}$$

Therefore the midpoint roll angle for the input mesh is:

$$\lambda_1 = \frac{C_1 \sin \eta_1 - \sqrt{r_{A1}^2 - r_1^2} + \sqrt{r_{A2}^2 - r_2^2}}{2 r_2} \tag{A.3}$$

Where

λ_1 is the mean roll angle between LPSTC and HPSTC with gear 1;
C_1 is the length of the line of centers between the driving gear and the idler;
η_1 is the working pressure angle at the input mesh;
r_{Ai} is the addendum radius of gear I;
r_i is the base circle radius of gear I;
z_2 is the number of teeth on the idler gear.

Making use of Figure A.1 we see that:

$$\sigma_1^L = \Phi_1 - \eta_1 + \lambda_1 \tag{A.4}$$

Where

σ_1^L identifies the origin of involute (OI) of the left flank in contact with the driving gear;
Φ_1 describes the location of the driving (input) gear relative to the idler. Similarly, the mean roll angle between the LPSTC and HPSTC with gear 3 is:

$$\lambda_3 = \frac{C_3 \sin \eta_3 - \sqrt{r_{A3}^2 - r_3^2} + \sqrt{r_{A2}^2 - r_2^2}}{2 r_2} \tag{A.5}$$

Where

λ_3 is the mean roll angle between LPSTC and HPSTC with gear 3;
C_3 is the length of the line of centers between the idler and the driven gear;
η_3 is the working pressure angle at the output mesh.

Making use of Figure A.2 we see that:

$$\sigma_2^R = \Phi_3 + \eta_3 - \lambda_3 - 2\pi \tag{A.6}$$

Where

σ_2^R identifies the OI of the right flank in contact with the driven gear;
Φ_3 describes the location of the driven (output) gear

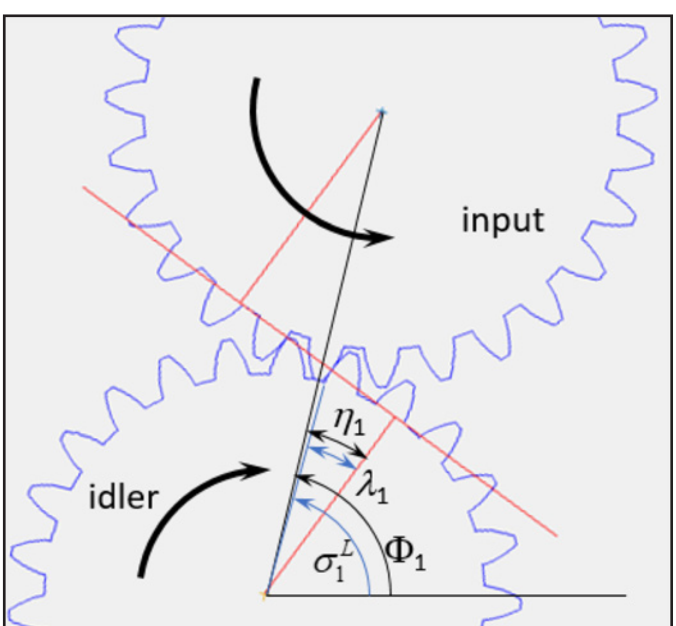

Figure A.1 Idler in mesh with input gear.

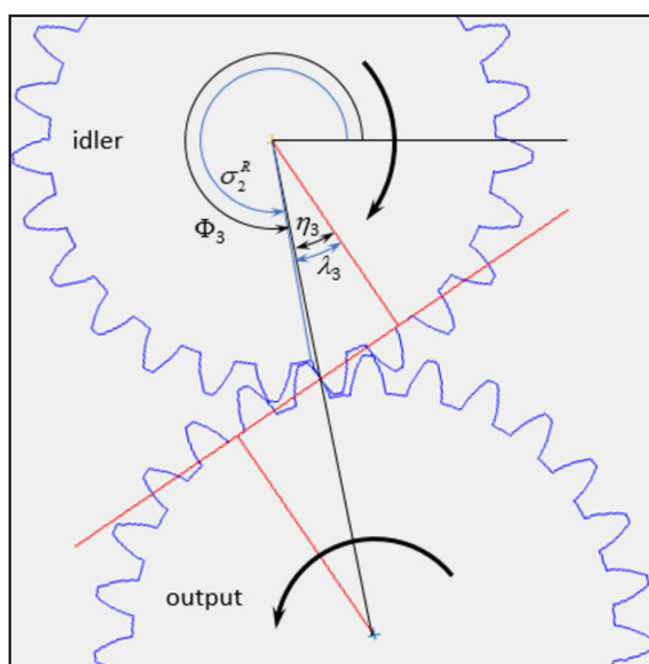

Figure A.2 Idler in mesh with output gear.

relative to the idler.

For idler rotation in the clockwise direction, 2π is subtracted so that σ_2^R is less than σ_1^L.

To go from the origin of involute on the right flank to the origin of involute on the left flank utilizes the tooth thickness.

$$\sigma_2^L = \sigma_2^R + 2\tan\varphi_t - 2\varphi_t + \frac{t_w \cos\varphi_t}{r_2} \tag{A.7}$$

Where

σ_2^L identifies the OI of the left flank of the tooth in contact with the driven gear;

φ_t is the transverse pressure angle of the hob;

t_w is the tooth thickness.

The sum of the last three terms of equation A.7 is the angle subtended by the tooth thickness at the base circle.

The number of teeth that are indexed to rotate the gear from σ_1^L is σ_2^L:

$$N = \frac{z_2}{2\pi}(\sigma_1^L - \sigma_2^L) \tag{A.8}$$

When N is an integer, both meshes occur simultaneously and are perfectly in phase. A phase shift in degrees is obtained by multiplying the mantissa of N by 360.

The ellipse is then generated as the sum of the transmission error force vectors at two lines of action.

$$\boldsymbol{F} = F_1 \cos(2\pi m f_1 t)\, e^{i\theta 1} + F_2 \cos\left(2\pi m f_1\left(\text{t} - \frac{\varphi}{2\pi f_1}\right)\right) e^{i\theta_2} \tag{A.9}$$

Where

$\boldsymbol{F}$ is complex notation for the coordinates of the ellipse;

F_i is the magnitude of the TE force for line of action i;

f_1 is the fundamental mesh frequency;

t is the time sweeping over one mesh cycle, going from 0 to $1/f_1$;

m is the harmonic number;

θ_i is the angle of orientation of line of action i;

φ is the relative phase between the two idler meshes.

Except for Figure 12, all ellipses assume $F_1 = F_2 = 1$.

For the case where $F_1 = F_2$, the ellipse extremes are realized at $2\pi m f_1 t = m\varphi/2$ and $m\varphi/2 + \pi/2$. The major and minor axis lengths are twice the magnitude of F at these extremes. Inserting $2\pi m f_1 t = m\varphi/2$ into Eq. A.9 produces the axis dimension, a of Figure 9, oriented an angle $(\theta_1 + \theta_2)/2$. Putting $2\pi m f_1 t = m\varphi/2 + \pi/2$ into the same equation produces b, the length of the perpendicular axis.

$$a = 2\sqrt{2}\cos\left(\frac{m\varphi}{2}\right)\sqrt{1 + \cos(\theta_2 - \theta_1)} \tag{A.10}$$

$$b = 2\sqrt{2}\sin\left(\frac{m\varphi}{2}\right)\sqrt{1 - \cos(\theta_2 - \theta_1)} \tag{A.11}$$

And

$$\sqrt{a^2 + b^2} = 2\sqrt{2}\sqrt{1 + \cos(m\varphi)\cos(\theta_2 - \theta_1)} \tag{11.12}$$

Where

a is length of the axis of the ellipse at an angle of $(\theta_1 + \theta_2)/2$ (Fig. 9);

b is the length of the axis perpendicular to the first.

Appendix B

This appendix shows that phasing the transmission error forces by half a tooth (180° phase shift) tends to cancel the odd harmonics of mesh frequency but double the amplitudes at the even harmonics. The TE force amplitudes at all harmonics are assumed to be unity.

Given two parallel unit force vectors with fundamental frequency f_1 and phase shifted by $\varphi = \pi$, their resultant force is:

$$F = \cos(2\pi f_1 t) + \cos(2\pi f_1 t - \varphi) \tag{B.1}$$

The phase shift can be written as a time shift by:

$$\cos(2\pi f_1 t - \varphi) = \cos(2\pi f_1 (t - \tau)) \tag{B.2}$$

Where

$$\tau = \frac{\varphi}{2\pi f_1} = \frac{\pi}{2\pi f_1} = \frac{1}{2f_1} = \frac{\text{T}}{2}$$

T is the fundamental tooth pass period.

The 180° phase shift is tantamount to shifting the waveform half a period. But the transmission error forces are made up of several harmonics.

$$F = \sum_{\text{m}}\left[\cos(2\pi m f_1 t) + \cos\left(2\pi m f_1\left(t - \frac{1}{2f_1}\right)\right)\right] \tag{B.3}$$

But

$$\begin{aligned}\cos\left(2\pi m f_1\left(t - \frac{1}{2f_1}\right)\right) &= \cos(2\pi m f_1 t)\cos(\pi m) + \sin(2\pi m f_1 t)\sin(\pi m)\\ &= \cos(2\pi m f_1 t)\cos(\pi m)\\ &= (-1)^m \cos(2\pi m f_1 t)\end{aligned} \tag{B.4}$$

Therefore

$$F = \sum_{\text{m}} \cos(2\pi m f_1 t) + (-1)^m \cos(2\pi m f_1 t) \tag{B.5}$$

The forces perfectly cancel when the two parallel transmission error force vectors are equal in magnitude and 180° out of phase and m = odd integers. However, the force sum is doubled for m = even integers. **PTE**

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