Experimental Determination of Oil Rheology Parameters to be Implemented in Power Loss Predictions of Gears and Rolling Element Bearings

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Introduction

Rheology models express the way tribological conditions translate to shear stress of the lubricant and friction force on the interacting surfaces. Due to the complexity of the lubricant rheology, the friction coefficient is usually obtained experimentally either under the same operating conditions or by curve fitting in a properly chosen friction map. The current study aims at determining the rheological parameters of a lubricant based on friction measurements carried out on a commercial, readily available ball-on-disc machine. They can then be implemented in power loss prediction methods that utilize state of the art thermo-elastohydrodynamic numerical models considering the non-Newtonian lubricant behaviour and the dependency on pressure and temperature of the lubricant properties.

Lubricants are commonly macro-molecular chains that behave like polymers in elastohydrodynamic lubrication. These chains follow a Newtonian linear law during steady state or shear rates close to zero. However, they deviate greatly under conditions of high pressure and high shear rate such as those in a typical gear mesh or a rolling element bearing. Under those conditions, the maximum friction typically peaks and reaches a plateau at around 0.08, far less compared to that a linear law would predict. Evans and Johnson (Ref. 1) using the extended rheology equation by Johnson and Tevaarwerk (Ref. 2) classified the behavior of the lubricant into four distinct regions, which indicate if it stems from the non-linear viscous or linear-elastic regime. The classification is based on the Deborah number defined as the ratio of the relaxation time of the lubricant to the time needed to pass through the contact. When it becomes greater than unity, which is typical for EHD contacts, the traction curve (friction coefficient over slide-to-roll ratio) is linear-elastic at first and then non-linear with a potential peak. The extended rheology equation uses a hyperbolic sinus function which is attributed to the studies of Eyring (Ref. 3) on polymers. The *sinh()* function is used to model the thermal activation theory of a molecule which defines the amount of work a molecule must perform to jump from an energy well to the next.

This aspect is known to be affected by both temperature and pressure and hence it is reasonable to expect a similar dependence in lubricants as well. Indeed, there are many different models proposed, such as those by Johnson and Tevaarwerk (Ref. 2), Houpert et al (Ref. 4), and Mihailidis and Panagiotidis (Ref. 5). Some contain both parameters while others omit temperature in favour of a simpler formulation.

Friction would still rise with the increase of shear rate even if the thermal influence on viscosity were negligible, due to the term. Limiting shear stress introduces a threshold to the maximum shear stress that a material could sustain before actually deforming as a "plastic" one. The theory was first proposed in 1960 by Smith (Ref. 6), although hinted in a previous work of Petrusevich in 1951 (Ref. 7). The flow mechanism of thermally activated zones and viscous flow has been shown in polymers to give its place to a different one in shear stress above $G/30$, where G is the shear modulus. The new mechanism is the formation of a shear band. A straightforward separation of the thermal effects due to shearing is almost impossible (Johnson and Greenwood (Ref. 8)). Experiments by Bair and Winer (Ref. 9) in low shear rates but very high pressures in an isothermal disc machine showed a clear and distinct maximum of the friction coefficient indicating a limiting shear stress. Further calculations and later microflow images of shear bands have been presented by Bair (Ref. 10). On the other hand, there have been additional phenomena observed, such as wall slip — especially in dissimilar, interfacing materials (Guo et al (Ref. 11)) that may also contribute to the reduction of the friction. Despite that, in steel on steel friction these phenomena have been only observed under extreme sliding, thus the limiting shear stress can arguably be considered the most probable explanation.

Various models for implementing in simulation the theory of the limiting shear stress have been proposed. Initially, Bair (Ref. 12) attempted an analytic approach in order to develop a parameter that would contain physical properties such as bond strength of the molecular structure. The calculation of such parameter is quite difficult and in practice it was experimentally obtained. In fact, this is an issue, since the temperature effect on the oil viscosity under high pressure is very difficult to isolate and calculate outside of an EHL contact. The second issue relates to the fact that an EHL contact is not at constant pressure overall, which in turn means that the total friction force is a sum over the contact area that includes both thermally activated zones and shear bands. Temperature and pressure have a strong impact on the shear band formation by affecting the shear modulus. Houpert (Ref. 4) presented in his dissertation an exponential model concerning the effect of temperature on the limiting shear stress. Wang and Zhang in 1987 (Ref. 13) also utilized an exponential law, which was later modified by Hsiao and Hamrock in 1992

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(Ref. 14). Roshetov and Gryazon, as mentioned by Wikström and Höglund (Ref. 15), presented an equation that includes first degree factors concerning pressure and temperature, which are multiplied. Kleemola and Lehtovaara in 2008 (Ref. 16) presented two models, a simplified and a more complex multi-parametric one including both second order and exponential laws. The present study incorporates a simple relationship to model the influence of temperature and pressure without exponential function that could generate instability in a solver.

The present study describes an experimental-analytical procedure that has been developed, in order determine the Eyring stress and the limiting shear stress of a given oil as functions of temperature and pressure. Special care has been taken to use commercially available equipment and standard experimental procedures. These data could then be used as input to any EHL model that would require them in order to calculate the friction coefficient accurately. As an example, the rheological parameters of the FVA (Forschungsvereinigung Antriebstechnik) reference oil Nr. 4 are determined.

Rheology Parameter Extraction Workflow

The lubricant rheology can be described using two equations.

- General rheology (Eyring) equation that requires two additional models
  - $\tau_E(p, T)$ Eyring stress
  - $\tau_L(p, T)$ Limiting shear stress
- Lubricant viscosity $\eta(p, T)$

The general rheology equation (Eq. 1) incorporates two terms — the linear-elastic and the non-linear viscous.

$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v = \frac{1}{G}\frac{d\tau}{dt} + \frac{\tau_E}{\eta} sinh\left(\frac{\tau}{\tau_E}\right) \quad (1)$$

For the EHD contacts considered, such as those found in gears and rolling element bearings the first term of Equation 1 can be neglected (Ref. 1).

The Eyring stress included in the above equation is affected by pressure and temperature. Many models have been proposed (Table 1).

Table 1 Eyring stress models including temperature and pressure effects

Johnson & Tevaarwerk	$\tau_E(p) = \tau_{E0}(1 + a_\tau p)$	(2)
Houpert et al	$\tau_E(p, T) = (a_\tau p + \tau_{E0})e^{\beta_\tau\left(\frac{1}{T} - \frac{1}{T_o}\right)}$	(3)
Mihailidis & Panagiotidis	$\tau_E(p, T) = \tau_{E0}(1 + a_\tau p)(1 + \beta_\tau(T - T_0))$	(4)
Present study	$\tau_E(p, T) = \tau_{E0}(1 + a_\tau p) + (\beta_\tau(T - T_0))$	(5)

Equation 1 is valid up to the shear rate, where the limiting shear stress is reached. It is also a function of temperature and pressure, for which the models in Table 2 have been proposed.

Table 2 Limiting shear stress models, including temperature and pressure effects

Bair	$\tau_L(p) = \Lambda p$	(6)
Johnson & Tevaarwerk	$\tau_L(p) = c_0 + c_1 p$	(7)
Houpert	$\tau_L(p, T) = \tau_{L0} \exp\left(a_{\tau L} p + \beta_{\tau L}\left(\frac{1}{T} - \frac{1}{T_o}\right)\right)$	(8)
Houpert et al	$\tau_L(p, T) = (0.015p - 1.88 * 10^{-3})\, e^{\left[585\left(\frac{1}{T_m + 273} - \frac{1}{+313}\right)\right]}$	(9)
Hsiao & Hamrock	$\tau_L(p, T_m) = \tau_0 E' + \gamma p)\, e^{\left[B\left(\frac{1}{T_m + 273} - \frac{1}{T_0 + 273}\right)\right]}$	(10)
Bair	$\tau_L(p) = c_1 p(1 + \beta_{\tau L}(T - T_0))$	(11)
Zhang & Wen	$\tau_L(p) = \begin{cases} \tau_{L0} & p < p_s \\ \tau_{L0} + a_{\tau L}(p - p_s) & p > p_s \end{cases}$	(12)
Kleemola & Lehtovaara	$\tau_L(p) = c_1 p + c_2 p^2$	(13)
Kleemola & Lehtovaara	$\tau_L(p, T) = (\tau_0 + \alpha_1 p + \alpha_2 p^2)(\alpha_3 - (\alpha_4(T - T_0)^{\alpha_5}))$	(14)
Lohner et al	$\tau_L(p, T, v_\Sigma) = \frac{4}{\pi}\left(a_1 p_m + a_2 + a_3 \lambda n\left(v_\Sigma * 1\frac{s}{m}\right) + a_4 T_0\right) \sinh\left(\frac{T_0}{T}\right)$	(15)
Present study	$\tau_L(p, T) = (c_1 p + c_2 p^2)(1 + \beta_{\tau L}(T - T_0))$	(16)

The present study proposes the use of the models described by Equations 5 and 16. Equation 5 is proposed since, disassociating the pressure from the temperature influence, results in a more gradual increase of the Eyring stress closer to that observed by Johnson and Tevaarwerk (Ref. 2) and the findings of the present study as well. The use of Equation 16 is proposed due to the observed measurements and the need for a simpler model.

The workflow presented will lead to the calculation of 6 parameters, i.e. — 3 for each of these equations.

Calculating the correct Eyring stress depends on having a correct estimation of the viscosity. According to Evans and Johnson (Ref. 1), using the rheology law, the oil viscosity values could also be obtained if the Eyring stress is known. In reality, the Eyring stress is not known while viscosity may be known within a rather limited pressure and temperature range. For each experiment, an Eyring stress and a viscosity value can be actually determined by fitting the rheology law in the region where the lubricant behavior is non-Newtonian. This is observed as a straight line in semi-log plot of the friction coefficient over the shear rate. The viscosity value at the mean pressure calculated according to a viscosity model can also be used and compared. Comparing the four most commonly used viscosity equations (Roeland (Ref. 17), Rodermund (Ref. 18), Doolittle (Ref. 19) and Shilling (Ref. 20)) showed huge differences in the predicted viscosity. This issue arises because the parameters are estimated from measurements under 200 MPa and extrapolated to 1 GPa. Using the rheology law to estimate the viscosity at higher pressure and comparing the values to those extrapolated from measured viscosity data at pressures less than 200 MPa showed that, the best fit was given by the equations of Roeland (17) and Rodermund (18), the latter of which is used in the current study (Eq. 17).

$$\eta(p, T) = \mathrm{A} * \exp\left(\frac{\mathrm{B}}{T + C - 273}\left(1 + \frac{p}{2 \cdot 10^8}\right)^{\mathrm{D} + \mathrm{E}\frac{\mathrm{B}}{T + C - 273}}\right) \quad (17)$$

where:

$\eta\left[\frac{N}{ms}\right]$: viscosity; $T[K]$: temperature; $p[Pa]$. pressure and

$\mathrm{A}\left[\frac{N}{ms}\right]$; B[K]; C[K]; D[–]; E[–]: parameters

The extraction workflow proposed is composed of four steps. Firstly, the experimental conditions have to be identified for nine experiments. Secondly, the conditions selected are run in a suitable ball on disc machine capable of as close as possible to isothermal testing. Thirdly, the calculation algorithm for the Eyring stress at each test point is performed. Finally, using these values, the parameters for the models of τ_0 and τ_L are extracted. These steps have to be performed once for the Eyring equation (Eq. 7) and once for the limiting shear stress (Eq. 16). Ideal selection of the test conditions can allow for the process to be run only once.

Experimental conditions and test rig. The Eyring stress is extracted from friction measurements. But, utilizing measured friction coefficient values to extract parameters that will be later used in EHL models to calculate the friction coefficient may result in a logical loop, which must be avoided. The present work does so by discarding the experiments included in the workflow of parameter extraction from any further validation or comparison. Only significantly different conditions or different test rigs, contact geometries etc. can be examined and simulated for validation of the previously obtained parameters.

In a ball-on-disc machine, the shear heating can be limited by setting the rolling speed and the normal force low. Provided that the geometry of the specimens is properly chosen, the contact pressure can be sufficiently high. In this way, quasi isothermal test conditions are maintained enabling thus the calculation of the Eyring stress.

The first step of the workflow is to determine the experimental conditions. The calculation process requires a set of nine traction curves which are spread across a typical temperature range such as 50 to 110°C, and across a pressure range within the specifications of the test rig — typically 0.5 to 1.25 GPa. The maximum-selectable temperature and pressure depends on the viscosity of the lubricant under evaluation and the calculated central film thickness. The method is limited to pure EHL under fully flooded conditions, since no surface interaction is considered. After an evaluation of the three most commonly used film thickness equations (Hamrock et al (Ref. 21), Chittenden (Ref. 22), Moes (Ref. 23)) and tests on an EHD2 (*The EHD2 machine is a ball-on-disc machine that utilizes a semi-transparent chrome-coated glass disc allowing optical interferometer measurements.*) machine (Fig. 1), the first two provided the best approximation for circular contacts when multiplied by the thermal parameter C_t, introduced by Gupta et al (Ref. 24). As the conditions used for the parameter extraction, the experiments are quasi-isothermal and, therefore, the equation of Hamrock et al (Eq. 18) provides sufficient accuracy without the thermal parameter

$$H_c = \frac{h_c}{R_x} = 2.69 U^{0.67} G^{0.53} W^{-0.067} \left(1 - 0.61 e^{-0.73\mathrm{k}}\right) \tag{18}$$

where

$$W = \frac{F_N}{E' R_x^2};\ U = \frac{\eta_0 V_x}{E' R_x};\ H = \frac{h}{R_x};\ G = aE';\ \mathrm{k} = \left(\frac{R_y}{R_x}\right)^{\frac{2}{\pi}} \text{and } V_x = \frac{V_{x1} + V_{x2}}{2}$$

The selected slide-to-roll-ratio (SRR) range should contain all three regimes of a traction curve. Typically, a maximum value 50% should be sufficient as to not increase the temperature significantly or risk mixed lubrication occurring. The required number of SRR settings should be quite dense near the low values, with roughly 30 points being sufficient to cover the whole range. The rolling speed selection requires calculation of the maximum temperature increase (at the contact to ensure that, it is less than +2 degrees. Calculating the limiting shear stress is not straightforward because the tests where the limit is reached must be identified first. In order to observe any limit occurring, higher pressures and speeds may be required. It is possible, that certain high viscosity oils may prove difficult to attain this condition without significant shear heating. Ideally, a clear maximum is needed at a rather low slide to roll ratio (or slip). Values in the range of 0.5 to 2 m/s for lubricants with ISO VG 460 to 100 may be used. Using the previous calculations, 9 condition sets for temperature, pressure and a common speed for calculating the Eyring stress model parameters, as well as another nine conditions sets for the limiting shear stress evaluations are defined. Repeat experiments may be run to verify the error margin, but generally for fully flooded EHL conditions it is low.

Eyring stress. Having experimentally obtained the friction coefficient vs. SRR curves for the selected conditions, the parameters for the Eyring stress equations are extracted in an automated way using software developed in the L.M.E.M.D (*Laboratory of Machine Elements and Machine Design, Aristotle University of Thessaloniki*). The algorithm is presented below.

At first, for each experimental point, the mean film thickness and the resulting mean shear rate are calculated using Eq. 19.

$$\dot{\gamma}_m = \frac{V_{x1} - V_{x2}}{h_m}, \text{where } h_m \approx h_m \tag{19}$$

Then, for each given set of experimental conditions, namely temperature and mean pressure, the resulting friction coefficient versus mean shear rate diagram is considered. In a semi-logarithmic plot, such as the one shown (Fig. 2), it is typically composed of three discrete sections: an initial almost flat section, a second quite pronounced linear with a constant slope and a final third non-linear that has varying slope as the shear rate increases. The second linear section is used to calculate the Eyring stress by numerically solving the following equation:

$$\dot{\gamma}_m = \frac{\tau_E}{\eta_{p_m}} \sinh \frac{\mu p_m}{\tau_E} \tag{20}$$

It is derived from the general rheology equation (Eq. 1) when the linear-elastic term is neglected and the shear stress τ

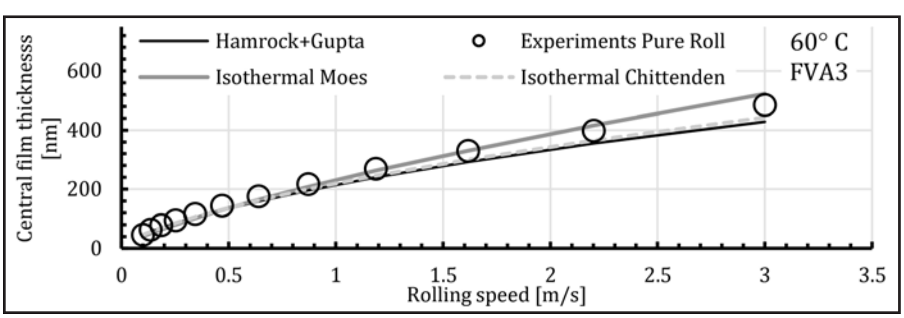

Figure 1 Comparison of the calculated and measured central film thickness.

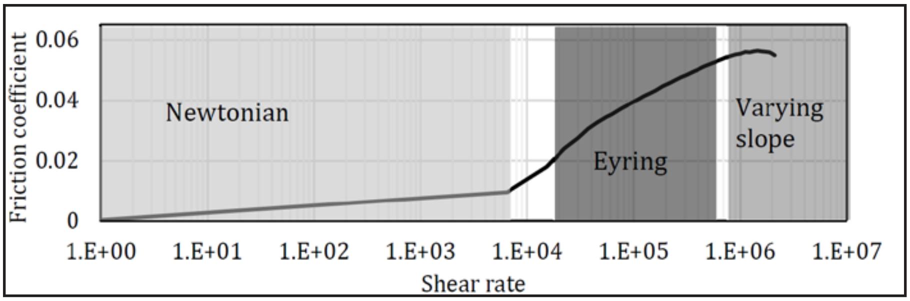

Figure 2 Coefficient of friction on a semi-logarithmic diagram versus shear rate presenting three main sections.

substituted by μp_m. For convenience, the following form of Eq. 20 can be used with sufficient accuracy, which results by expanding the *sinh()* function in Taylor series:

$$\frac{\tau_m}{p_m}\ln\left(\dot{\gamma}_m\frac{\eta_{pm}}{\tau_E}+\sqrt{\left(\dot{\gamma}_m\frac{\eta_{pm}}{\tau_E}\right)^2+1}\right)=\mu \tag{21}$$

Finally, the parameters of Equation 5 are calculated by using the resulting Eyring stress of all testing conditions.

Limiting shear stress. The point, where the maximum friction occurs is used to determine the limiting shear stress under the corresponding temperature and pressure.

$$\tau_L=\mu_{max}p_m \tag{22}$$

Then the parameters of Equation 16 are obtained by curve fitting to the limiting stress values obtained from all testing conditions.

This is an approximation, since the measured friction coefficient is mainly a result of the shear stress created in the high-pressure area of the contact where the oil has reached its shear stress limit. The contact area encompasses some very low pressures near the edge, that do not contribute significantly to the shear stress or the mean pressure integral.

If a clear maximum is not obtainable under the selected conditions, higher pressures may be required resulting in a exceeding 2 degrees. In such case, the limiting shear stress is assumed at the mean contact temperature.

EHL Model

Aiming to compare the measured friction coefficient against calculated values, an EHL model is used (Mihailidis et al (Ref. 25)). It combines non-Newtonian lubricant behavior, thermal effects, as well as the influence of pressure and temperature on the thermal properties of the lubricant. The solution is performed in a multi-grid multi-level manner. For the present study the local EHL pressure spike is of small interest so the starting grid is limited to 25 × 25 nodes symmetrical around the oil entrainment axis, and the depth is limited to 3 levels resulting into a 100 × 100 fine mesh grid. A line relaxation of the Reynolds equation is used, while the convergence criteria are limited to 6e-4 for the pressure and 2e-3 for the temperature. The solution space is limited from -3.5 to 2.5 in the X axis, and -2.5 to 2.5 for the Y axis. The temperature field in the Z axis is composed of 9 nodes in the film equally spaced, and 5 nodes in the contacting surfaces with a geometrically increased spacing where the first two nodes are spaced equally to those in the film for convergence reasons. The numerical solution of EHL requires adjustment of solution process due to abrupt changes in pressure as well as due to the mutual dependence of oil characteristics on temperature and pressure. The initial film thickness estimation plays an important role in the convergence. The selection of its value is based on the equations of Kudish (Ref. 26). The use of the limiting shear stress model creates a nonlinear abrupt change in the behavior of the film. The method for stabilizing the solution is to limit the initial pressure converge cycle to 1 loop and allow the thermal convergence to be reached. The pressure loop begins by assuming a Hertzian distribution of the pressure and adjusts it. Additionally, a limit of the maximum pressure that can be present at a grid point limits the effects of single point singularity occurring due to grid size. In cases with very high film thickness, the relaxation factor of the film height is reduced below 1E-2 even down to 1E-4. This increases the number of loops but significantly improves stability.

Results

The proposed methodology is developed to extract the parameters for the relationships and of a non-Newtonian lubricant. As a first application, the corresponding parameters of the FVA 4 reference oil are determined. The measurements were carried out on an MTM machine located of the tribology group of Imperial College in London, UK. Such ball-on-disc machines are commercially available, manufactured by PCS Instruments and sold under the name Mini Traction Machine 2 (Ref. 27). The friction coefficient vs. shear rate curves are shown in the semi-logarithmic plot (Fig. 3). The selected experimental conditions for are 0.5 m/s rolling speed, 0.7, 0.81, 0.9358 GPa maximum pressure and 60, 75, 90º C temperature. For the limiting shear stress , experimental conditions were 1 m/s rolling speed, maximum pressure 0.9358, 0.985 and 1.1 GPa at the same temperatures, resulting in a similar plot.

Since Equation 5 only has three parameters — a curve-fitting process is performed using the nine mentioned experiments. It can be said that parameters could be obtained with less experiments (namely 4), but since the data will be used for the limiting shear stress as well, nine experiments are required. The correlation factor is > 99%. The resulting parameters for FVA4 are the following:

Table 3 Eyring values for FVA4 oil

τ_{E0}	a_τ	β_τ	T_0	R^2
8.33 E5	2.82 E-9	1.647 E-2	273	0.993

Using the nine experiments, the same number of maximum friction coefficient values are obtained. Those are used to fit the proposed model of Equation 16. The correlation is over 96%. The resulting parameters for FVA4 are given below.

Table 4 Limiting shear stress equation parameters for FVA4 oil

c_1	c_2	$\beta_{\tau L}$	T_0	R^2
9.966 E-3	6.782 E-11	−0.002483	273	0.96

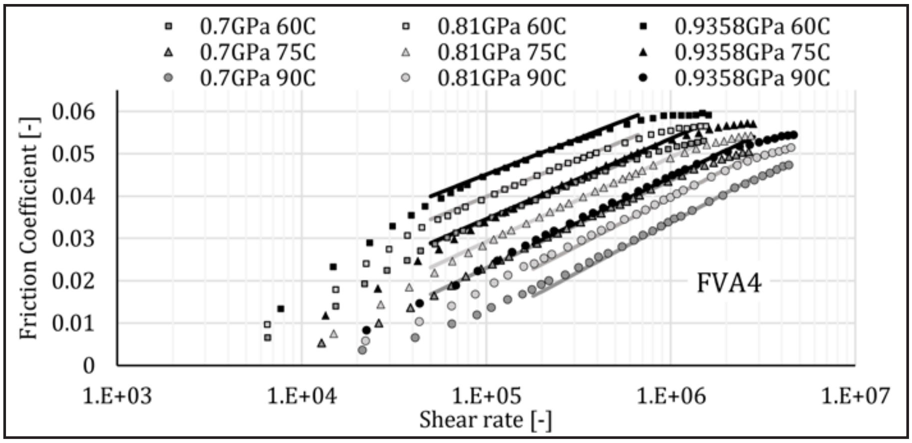

Figure 3 Friction coefficient vs log of shear rate. Data from the three different pressures along with fitting *sinh()* equation with the appropriate viscosity and Eyring stress for rolling speed 0.5 m/s.

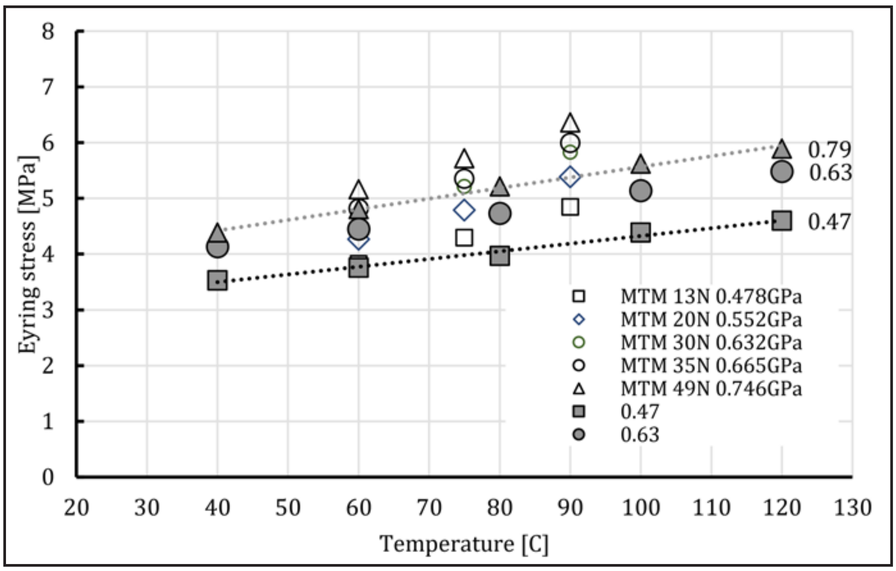

Figure 4 Calculated values from 15 experiments in an MTM (white markers) vs. values published (Ref. 1) for a similar viscosity oil at similar mean pressures (filled markers). Since not all tested temperatures are available an interpolation (dotted lines) for the in-between values is necessary.

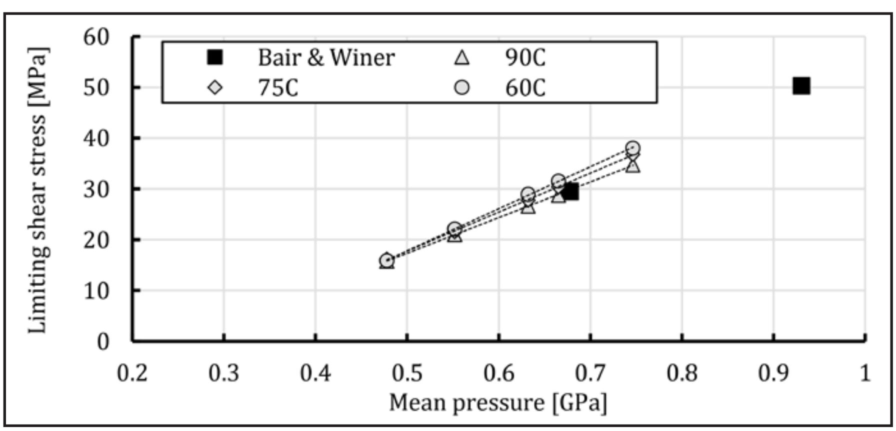

Figure 5 Comparison of calculated values of limiting shear stress for FVA4 vs. Bair and Winer for HVI650; Bair and Winer claimed that temperature does not influence the limiting shear stress, thus only one value is shown per pressure.

Preliminary Validation

In order to validate the results, three separate methods of comparison are performed.

- Comparison of the calculated Eyring stress was performed vs. published data for similar viscosity oils. The calculated Eyring stress values are acceptable since compared to the findings from Evans and Johnson (Ref. 1), they are within the same range (Fig. 4).
- Comparison of the limiting shear stress vs. published data from a ball-on-disc and two-disc machines. The method leads to values that are comparable for similar viscosity oils, as those reported by Bair and Winer (Fig. 5). Previous results by Evans and Johnson (Ref. 1) concluded that no significant effect of the temperature is observed for an ISO 460-HVI 650 (same viscosity as FVA4), but the present study found such a relation.
- Using a larger dataset of 90 test runs with FVA 4, and three repeats each at 5 different pressures, and temperatures from which conditions and experiments used in the methodology are excluded (speeds < 1m/s). For those experiments a simulation was set up in the thermo-EHL solver for calculating the friction force. The results are within a good accuracy ±–5% (Fig. 6).

For more information.

Questions or comments regarding this paper? Contact Emmanouil Athanasopoulos at *manos@feaqus.com*.

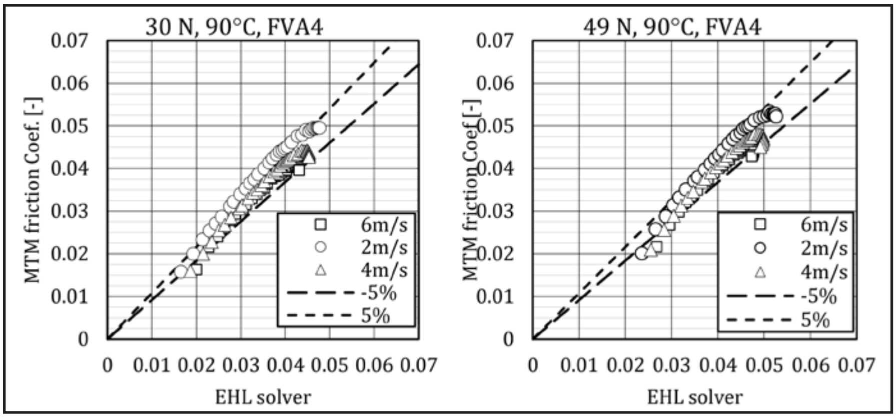

Figure 6 Comparison of the friction coef. measured in the MTM and the calculated values obtained using the Thermal-EHL solver.

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Conclusions

The method, outlined in the present study, achieved to extract the rheological parameters needed to describe the oil behavior in elastohydrodynamic contacts. The Eyring and the limiting shear stress, as well as the factors considering the temperature and pressure influence, are obtained by evaluating the friction coefficient measurements conducted under nearly isothermal, fully-flooded EHL conditions.

Based on the friction coefficient over slide-to-roll ratio measurements, obtained from 18 test runs following the proposed procedure, the rheology parameters for the FVA4 reference oil were extracted. They can be used in advanced EHL models to calculate the friction coefficient.

As a preliminary validation of the method, these parameters were then fed in a thermo-EHL solver and the friction coefficient was calculated. The results showed very good agreement with measurements carried out under conditions outside the range of those used to extract the rheology parameters. A final validation incorporating experiments on a two-disk machine is on the way.

Summing up, the proposed method allows the use of a commercial, readily available test rig with automated process and minimum oil requirements to extract rheology parameters needed for advanced thermos-EHL simulation models and for conditions commonly observed in rolling element bearing and gears. **PTE**

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