Mathematical Modeling for the Design of Spiroid, Helical, Spiral Bevel and Worm Gears

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Introduction

Spiroid and worm gears have superior advantages for high-torque and miniaturization applications. And for this reason they are particularly preferred in aerospace, robotic and medical applications. They are typically manufactured by hobbing technology, a process with a typical overall lead time of 4 to 14 weeks.

Besides the relatively lengthy lead time, and despite the fact that the tooth profile is defined through its pressure angles, 3-D drawings of the gears cannot be produced. This is due to the difficulty of capturing the entire curvature of the gear face from the outside to the inside diameter. Because of this difficulty, 3-D quality control and FEA (finite element analysis) under load are difficult and must be accomplished through classical analysis that incorporate pinion bending stresses, gear tooth shear stresses and compressive stresses between pinion and gear teeth. Due to some of these challenges, these gears have been limited to niche status in the industry.

This paper presents a novel work for Spiroid and worm gears that mathematically calculates the gear tooth profile in terms of the geometry of the cutting tool (hob) and machining set-up. Because of their similarity, the work is also expanded to spiral bevel gears. We have developed software to plot the gear tooth when the parameters of the geometry of the tool and machining set-up are entered. The gear tooth shape can then be altered and optimized by manipulating the input parameters until a desired tooth profile is produced. In effect, the result will be designing the hob and machining set-up for best gear tooth profile on the computer. The result is generated gear tooth data that is entered into CAD software to generate a true 3-D model of the gear. The tool path will also be generated from the same data for CNC machining.

This mathematical modeling allows for direct CNC machining, rather than hobbing, and may reduce the prototype lead time from weeks to hours. The pinion can be designed in a similar process, and its tooth can be graphed inside the gear's groove, showing the contact points and the clearance between the two surfaces. This novel work has already resulted in the invention of a new gear type combining Spiroid and worm gear in a single gear driven by the same pinion. This provides a significant increase in torque capability.

Mathematical modeling is presented in this paper as a tool design to reduce the lead time and cost for designing Spiroid, worm and spiral bevel gears. Mathematical modeling is based on mathematically calculating the 3-D gear tooth profile in terms of the cutting tool (hob) geometry, the machining set-up, and the gear size (inside and outside diameters). Software has been developed to allow designers to enter values of these parameters and observe the resulting 3-D gear tooth profile. The designer can observe the resulting tooth profile on the computer and adjust the input parameter values to obtain the desired profile. The software also generates the profile in numerical *xyz* points, which is necessary to produce 3-D drawings of the gear, conduct direct quality control, and perform FEA under load. It was also demonstrated that mathematical modeling can be a tool for gear innovation. It has already resulted in the invention of a new gear type in that a Spiroid/worm hybrid combines a double Spiroid gear and a worm gear in one. It more than doubled the gear torque capability, in comparison to a single Spiroid gear—with a minimal increase in size or weight.

Spiroid Gears

Oliver Saari invented Spiroid gears in 1954 while working for ITW, and the ITW Spiroid division was created. (*Author's Note: Spiroid is a registered trademark of ITW; the views expressed herein are those of the author alone and do not neces-*

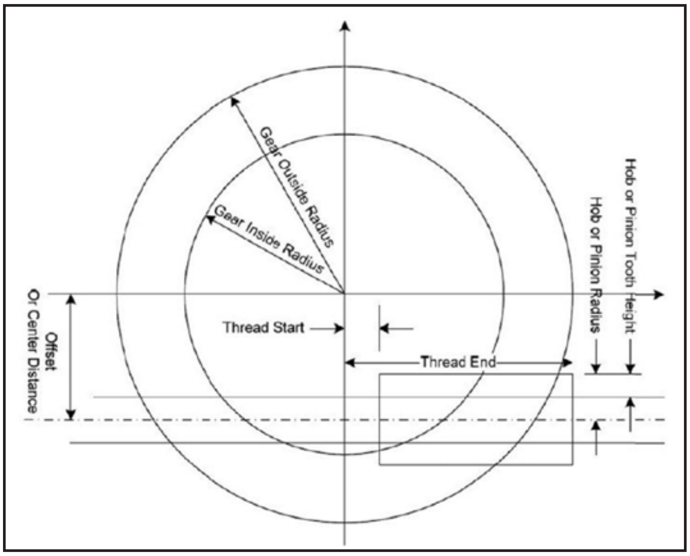

Figure 1 Spiroid gear/pinion assembly or gear/hob set-up.

Figure 2 The three types of Spiroid gear.

sarily reflect the views of ITW.) The gear set consists of a gear and a helical pinion assembled as shown in Figure 1. It is similar to spiral bevel gear set, except the pinion axis is moved a certain distance off the gear axis. This distance is called offset, or center distance. It makes it possible to hold the pinion shaft on both ends to increase leverage and stability. Spiroid gears are special gears with a wide range of RPM ratio, from as little as four to as many as several hundred; this facilitates single-stage designs that reduce size and increase efficiency. They can also have a high contact ratio, which makes them suitable for high-torque and low-noise requirements.

There are three Spiroid gear types (Fig. 2): flat-face; skewed angle with cylindrical pinion; and skewed angle with tapered pinion.

The pinion can be manufactured by any method for making helical gears, including grinding wheel, shaping, or CNC milling — but the grinding wheel is the dominant method. The gear is manufactured in a hobbing process, using a cutting tool (hob). The hob geometry is similar to the pinion geometry, except its tooth sides in the axial direction are straight (Fig. 3). It has a certain number of gashes in the direction perpendicular to the helix that are sharpened for cutting. During machining the gear and the hob will be rotating, with an RPM ratio equal to the ratio of their teeth numbers.

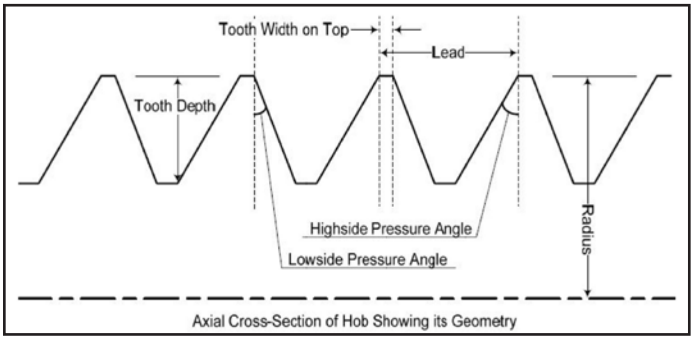

Figure 3 Axial cross-section of a cylindrical hob.

Figure 4 Cylindrical and tapered hobs.

A cylindrical hob and a tapered hob are shown in Figure 4. There are a total of nine basic geometry parameters; they are: outside diameter; tooth height; number of teeth (starts); lead (axial pitch); high-side pressure angle; low pressure angle; tooth width on top in axial direction; and the start and the end of its tooth along the shaft with respect to centerline. The taper angle is an additional parameter for the tapered hob. Other parameters that have to be taken into account when designing a hob are: gear inside radius; gear outside radius; gear number of teeth; and the center distance (off-set) between the gear axis and the hob axis. Also, in the skew angle case the skew angle is an additional parameter. This large number of parameters — 13–15 — makes it hard to predict the gear tooth shape and profile.

Designing a hob for a new gear set has the potential to be a trial-and-error process. The design objective is to have a gear tooth that is free of gouging on its sides or clipping of its top. It also should have the desired width and a balance of pressure angles, and land area at the root and at the top. After selecting the geometry the hob is manufactured by an outside machine shop with a lead time of eight to ten weeks. The gear is then cut in-house. It can be difficult during this process to realize the precise design objectives of the desired gear tooth geometry.

Initially the hob is placed over the gear blank (Fig. 1). During machining the gear and the hob will be rotating. Machining will be complete when the hob penetrates a distance equal to the tooth height. In the case of the skew angle gear, the hob axis will have an incline angle over the gear plane. After cutting a certain number of gears, the hob becomes dull; the hob will be sharpened and its diameter will be reduced by a few thousands of an inch each time. This will have some effect on the gear tooth profile and its contact with the pinion. Many manufactured gears are smaller than five inches in diameter.

Since the full-length tooth profile of the manufactured gear remains unknown, true 3-D models of the gear cannot be produced, and it can be difficult to determine FEA and tooth strength analysis under load outside of classical analytical methods.

Mathematical Modeling Solution

Gearometry has developed math equations and processes to accurately calculate the gear 3-D, *xyz*, tooth profile in terms of the hob geometry, the machining set-up, and the gear parameters. We also developed software in the form of *Excel* spreadsheets where values of the parameters are entered and the 3-D profile of the gear tooth or gear groove is produced in the form of sketches (Figs. 5–7) and in the form of *xyz* point data organized data arrays.

For gear tooth profile representation, the selection of an appropriate coordinate system is very important.

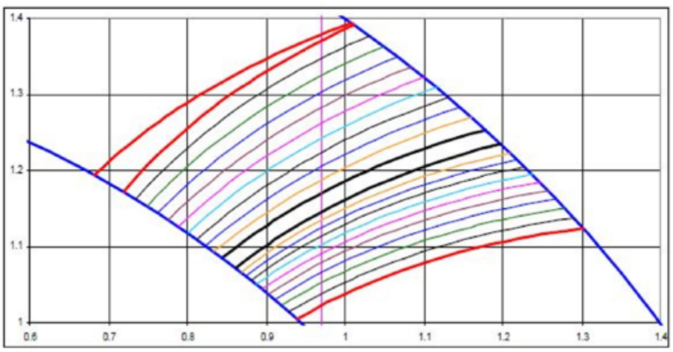

Figure 5 3-D sketch of an optimized, flat-face Spiroid gear groove — high and low sides.

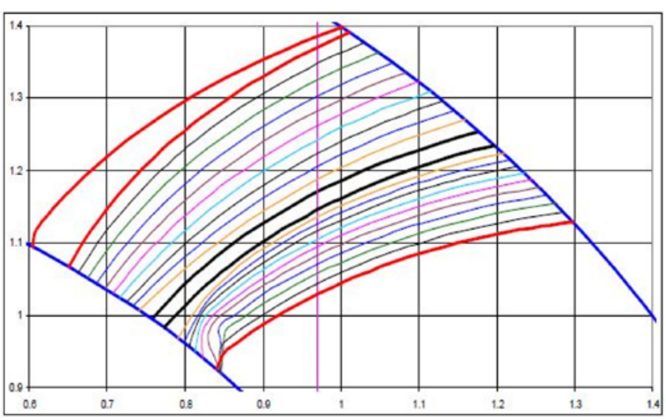

Figure 6 Gear groove profile for a gear inside radius of 1.25".

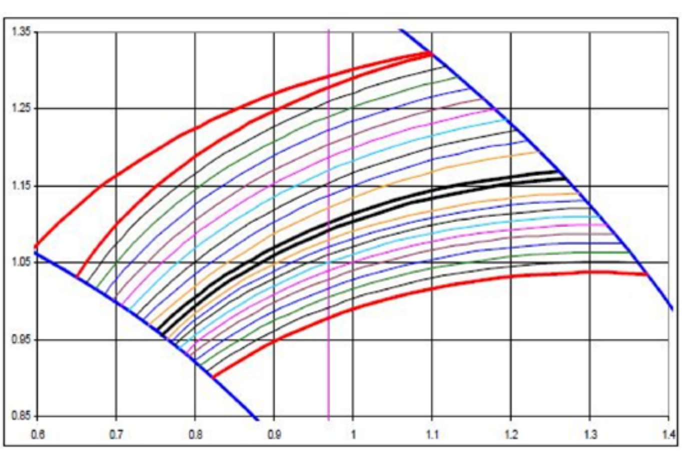

Figure 7 Gear groove profile with enhanced design using *Gearometry* software.

The z axis of our coordinate system is always aligned with the gear axis. For the flat-face gear, $z = 0$ is at the root of the tooth. The x axis is parallel to the center distance and the y axis is parallel to the pinion/hob axis. In the case of the flat-face gear, the xyz points of the tooth profile are arranged in 11 horizontal lines for the high side (parallel to the x-y plane), and 11 lines for the low side. Each line has 21 points. Therefore, each side has 231 points. The lowest line of each side is at the root where $z = 0$. The highest line is at the top where $z =$ tooth height. In Figures 5–7 the 11 lines in the southeast corner represent the low side, and the 11 lines in the northwest corner represent the high side. The two heavy black lines in the center are at the root, and the two heavy red lines are located at the tooth top. Therefore the sketches in these figures represent the gear groove where a tooth of a hob or pinion fits. The last heavy red line in the northwest corner is the top of the low-side of the next groove. The area between this line and the previous line is the land area of the tooth top. Each pair of lines of the same color represents a horizontal cross-section, in the x-y plane, of the gear groove. The cross-sections are equally spaced vertically. The separation distance between them is one-tenth of the tooth height.

As in conventional hob design, at the outset one enters values of the design parameters. But in our software the designer will be able to manipulate the entered values in the spread sheet and observe how the tooth profile will change until the desired design is obtained. This optimization process can typically take an hour or two, depending on individual cases and user satisfaction. Ultimately, this process will result in the designing of the hob geometry and determining of the optimum machining set-up parameters to produce the perfect gear.

For illustration we will explain the process of designing a set of Spiroid gear and its pinion. We will consider the flat-face gear set of Figure 2. This set was designed and manufactured in the conventional method. The values of its 13 design parameters are listed in Table 1. They were estimated using reverse engineering of the hob geometry from the available pinion geometry. We used our software to sketch the gear tooth profile with the parameters in Table 1 (also shown in Fig. 5). At this point we ask the question whether the gear tooth profile is truly optimized — or is it merely an example of an "acceptable defect-free" profile. When designing a gear set, the objective is to maximize the torque capability and contact ratio without increasing the set size. This means extending the gear inside radius as much as possible toward the center. Using our software, we

Table 1 Parameters and their optimized values of the flat-face gear design

Hob thread start	0.250"
Hob thread end	1.750"
Hob radius	0.410"
Hob tooth height	0.190"
Hob high pressure angle	40°
Hob low pressure angle	25°
Hob tooth width on top	0.025"
Hob number of teeth	4
Hob lead	1.500"
Gear inside radius	1.375"
Gear outside radius	1.720"
Gear number of teeth	27
Center distance	0.970"

extended the gear inside radius down to 1.25", from 1.375", while keeping all other parameters values the same. Figure 6 shows that the tooth profile begins displaying defects when the inside radius is below 1.30". However, the tooth will be defect free for inside radius of 1.31". Extending the gear inside radius down to 1.31" can significantly increase the contact ratio, increase the tooth strength and increase the gear set torque capability.

The drawing in Figure 7 shows that a perfect gear tooth profile can be obtained for an inside radius of 1.22" when the hob lead is reduced from 1.50" to 1.32"; the high-pressure angle is reduced from 40° to 35°; the low-pressure angle is reduced from 25° to 23°; and the tooth width on the top is reduced from 0.025" to 0.01". The gear outside radius and center distance, as well as the tooth height and the RPM ratio, are kept the same. With this new design the contact ratio is increased from 1 to 2, while the gear tooth is significantly increased in both circumferential and radial width.

However, this improvement is at the expense of the pinion tooth.

Using a narrower hob tooth, the pinion tooth width at tooth mid-height will be reduced by about 17.8%; but now we have two teeth in contact at any time rather than only one. Therefore the load-per-pinion-tooth is reduced by 50%. Depending on the application, the designer has to make decisions on whether to enhance the gear tooth, the pinion tooth, the contact ratio or some other component of the gear set. This software provides a wide range of options to the designer, with accurate information for decision making. In fact, additional good options could be further explored by manipulating the tooth height value. Smaller heights would allow wider groove or larger pressure angles. Both would lead to a wider pinion tooth.

In addition, the *xyz* data points of the curves in (Fig. 7) can be transported into engineering CAD software to generate the true 3-D model of the gear. One model is shown (Fig. 8) for a 35-tooth gear.

Such models can now be used to perform accurate FEA under load that will facilitate design decisions. Figure 9 shows FEA results on the Spiroid/worm hybrid gear (Fig. 21) where the 3-D model is generated from *xyz* data points.

The software also superimposes the hob tooth inside the machined gear groove. Figure 10 shows the gear groove of Figure 7 with the hob tooth superimposed inside it. The figure shows that the high side of the hob tooth is in contact with the gear tooth, along a line from a point at the bottom of the groove exactly below the hob axis, to a point at its rim and to the right of the hob axis. On the low side, the contact line also starts at a point at the bottom of the groove to a point at its rim, to the left of the hob axis.

The hob axis lies at $x = 0.97$". The hob and the gear can be incrementally rotated by a software command to show the tooth inside the groove in various positions. This enhances visualization of the pinion tooth and calculation of the contact ratio. In this case the contact ratio is 2; the conventionally designed gear has a contact ratio of 1.

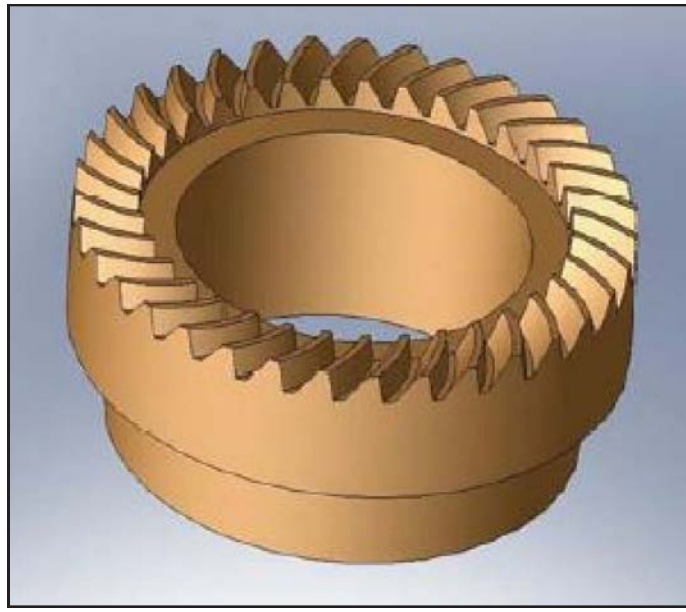

Figure 8 Example of true 3-D model of flat-face Spiroid gear from *Gearometry* software data.

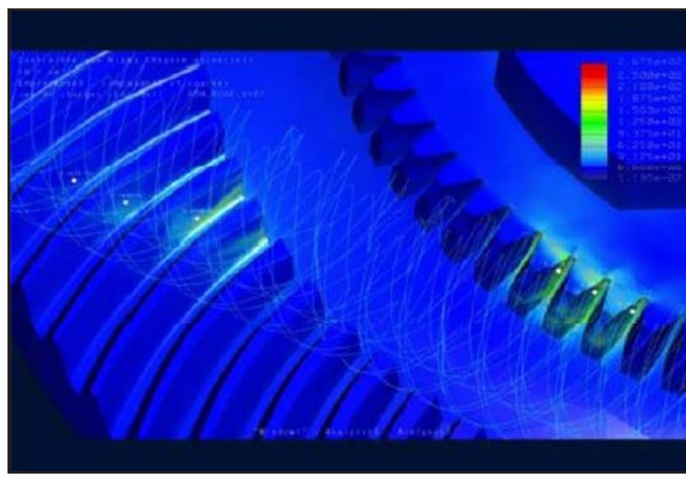

Figure 9 Example of FEA under load from *Gearometry* software data.

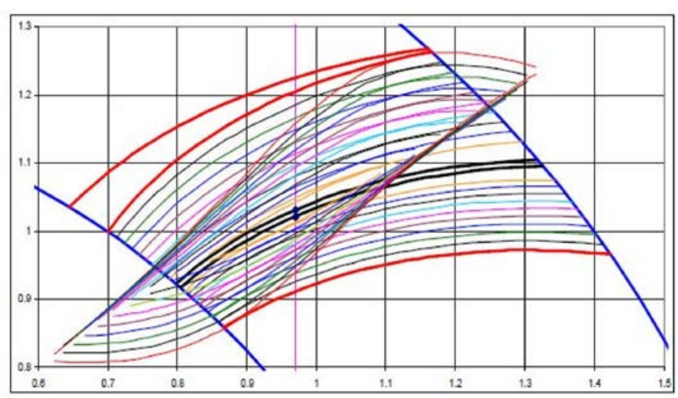

Figure 10 *Gearometry*-enhanced design gear groove with hob tooth superimposed.

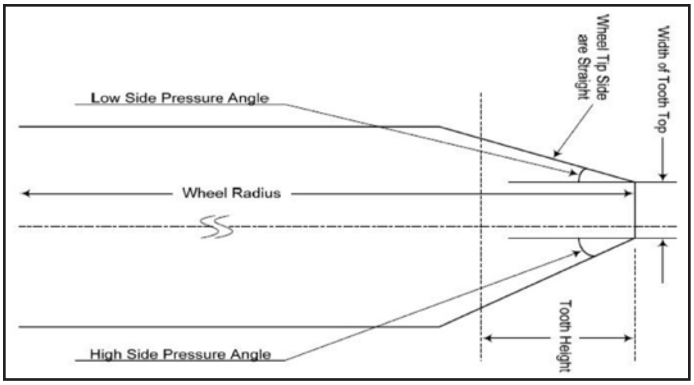

Figure 11 Grinding wheel geometry for Spiroid gear pinion.

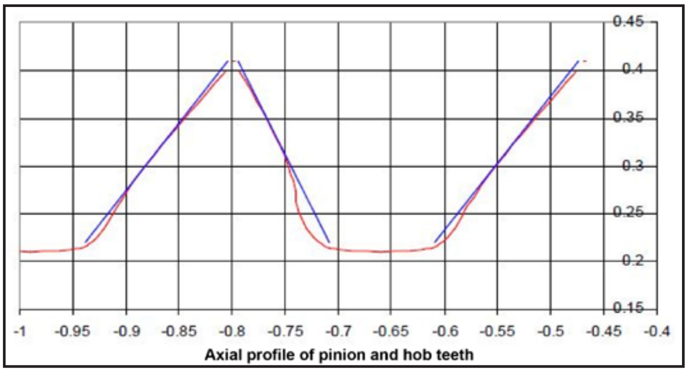

Figure 12 Pinion tooth (in red) inside the hob tooth (in blue).

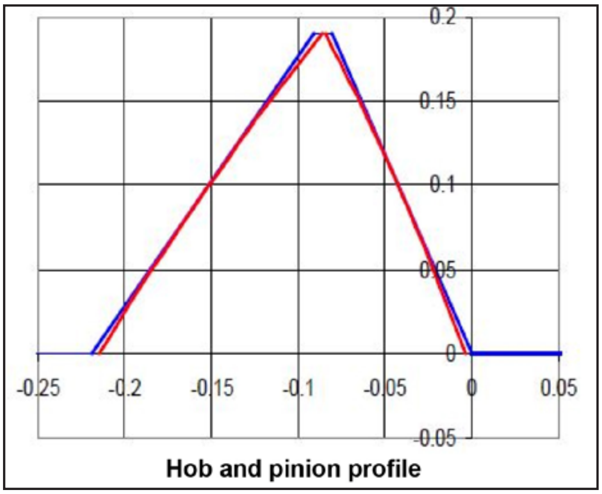

Figure 13 Pinion tooth (in red) with circular sides contained inside the hob tooth (in blue).

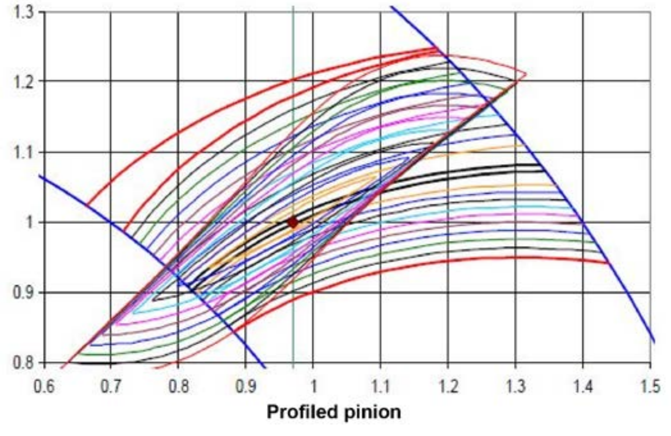

Figure 14 Circular sides pinion tooth superimposed inside the gear groove.

Spiroid Gear Pinion — A Helical Gear

Essentially, the pinion of the Spiroid gear is a helical gear. Thus our mathematical modeling of the pinion will be presented as modeling of the helical gear. In this paper we will design a pinion for the Spiroid flat-face gear. We will discuss two cases; the first is when the pinion is manufactured by a grinding wheel; the other is for using CNC milling, or shaping, by a tool. In any case, the objective is to design a pinion tooth that will be contained inside the hob tooth and touches the hob tooth in one point at about tooth mid-height (Figs. 12–13). In this case, its contact with the gear tooth under no load will be a single point on each side, at about a tooth mid-height.

Grinding Wheel

A typical grinding wheel is shown in Figure 11. Its geometry parameters are: the wheel radius; the high-side pressure angle; the low-side pressure angle; the tooth height; and the top tooth width. The other pinion design parameters are: the pinion outside radius; its number of teeth (starts); its lead; and the wheel tilt angle. We therefore have a total of nine parameters. Again, the profile of the pinion tooth is a function of these parameters. Figure 12 shows a *Gearometry*-optimized pinion tooth (in red) inside the hob tooth (in blue). The pinion and hob will have the same lead if they have the same number of teeth. There are cases where the number of teeth is different, in which their lead would change.

Another way of making a pinion is by CNC milling or shaping. This would only require the generation of the geometry of the pinion tooth. The data could then be entered in a CNC

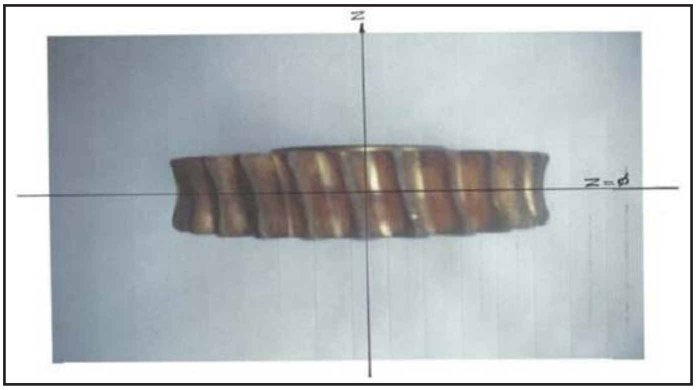

Figure 15 Single-enveloping worm gear.

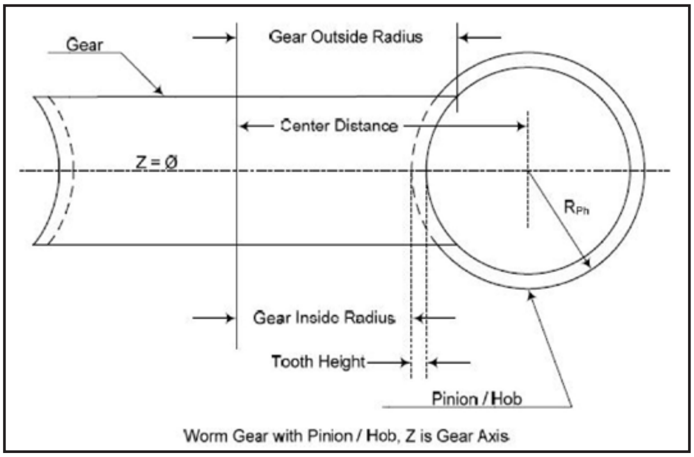

Figure 16 Worm gear/pinion assembly or gear/hob set-up.

machine for milling or used to generate the shaping-tool geometry. Figure 13 shows an axial cross-section of a pinion tooth (in red) contained inside the hob tooth (in blue). The red profile on each side of the tooth is a circular section that touches the blue profile at exactly mid-tooth height. *Gearometry* automatically produces the profile *xyz* point data and the sketch in (Fig. 13) to allow the designer to select the circular profile radius to control the clearance when moving away from the contact point.

Figure 14 shows a sketch of the pinion tooth of Figure 13 superimposed inside the gear groove of Figure 7, under no-load condition. They contact each other in a single point at mid-tooth height along the pink lines as shown. In fact, the software simplifies the calculation of the clearance between the gear groove and the pinion tooth when moving away from the contact point in vertical and horizontal directions. This aids in calculating the spreading of the teeth contact under load for a given material, and in conducting tooth strength and FE analysis on the gear set.

Worm gear. This paper discusses only the single enveloping worm gear. The pinion, or worm, is a cylindrical helical gear — similar to the flat-face Spiroid gear pinion. Like the flat-face Spiroid gear, the worm gear is typically manufactured in by the hobbing process, beginning with a cylindrical gear blank.

During machining the hob — with helical gear shape and sharp gashes — is placed over the cylinder surface. The hob axis is oriented in a perpendicular direction to the cylinder axis. The gear and the hob will be rotating with an RPM ratio equal to the ratio of the number of their teeth. Machining will be complete when the hob is advanced by a tooth height toward the gear center. The resulting gear will look like a helical spur gear with raised edges (Fig. 15); the raised edges will envelope the pinion for increased tooth contact.

The hob geometry and parameters are the same as that of a Spiroid gear hob. Therefore the total number of design parameters is 13. The hob geometry parameters are nine, and the other four are the gear inside radius; gear outside radius; gear number of teeth; and center distance (Fig. 16). (*Note that selecting non-optimized values for the parameters can result in either undercut of the tooth sides or clipping of the gear tooth on top*.)

Table 2 lists all 13 design parameters, along with their optimized values for a worm gear. In this case, the z axis of the coordinate system is aligned with the gear axis. Cross-sections of the gear tooth profile are presented in x-y planes perpendicular to the gear axis at different z values. Figure 17 shows a tooth cross-section in heavy pink at $z = 0$ mm, at exactly the gear center in axial direction where the tooth root is at the gear inside radius (in heavy red). For $z = 3$ mm, the tooth cross-section is shown in Figure 18.

Table 2 Design parameters with optimized values of a worm gear

Hob thread start	-10.0 mm
Hob thread end	10 mm
Hob radius	6 mm
Hob tooth height	2 mm
Hob high pressure angle	20°
Hob low pressure angle	20°
Hob tooth width on top	0.6 mm
Hob number of teeth	4
Hob lead	10 mm
Gear inside radius	28 mm
Gear outside radius	31 mm
Gear number of teeth	76
Center distance	34 mm
Tilt angle	10°

The software includes a new parameter in which the pinion axis is rotated around the centerline by a certain angle, i.e. — the "tilt angle." This adds a degree of versatility to the use of the worm gear, as will be demonstrated. The tooth has good proportions — from one end of the gear at $z = -3.5$ mm — to the other end at $z = 3.5$ mm. This is because the parameters were optimized. Figure 19 shows what the cross-section of the tooth would look like at $z = 0$ mm and when the hob lead is changed from 10 mm to 11 mm. It has steep sides

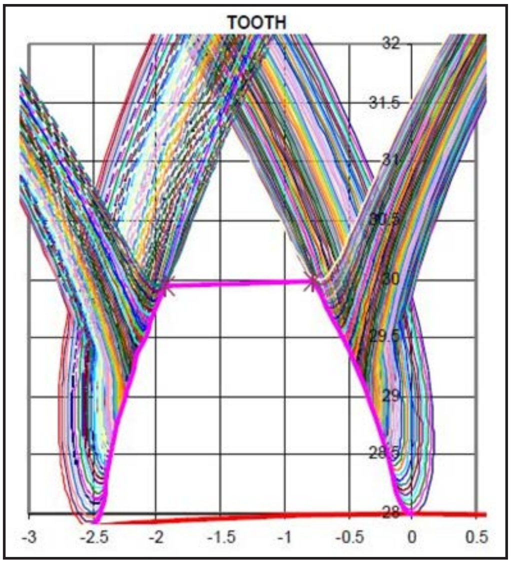

Figure 17 Axial cross-section of the gear tooth at $z = 0$ mm.

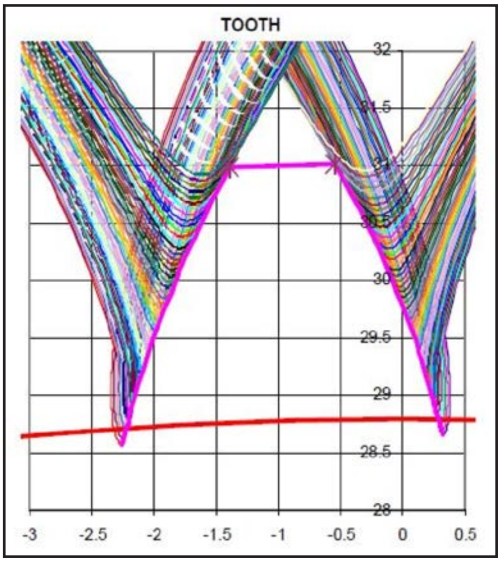

Figure 18 Axial cross-section of the gear tooth at $z = 3$ mm.

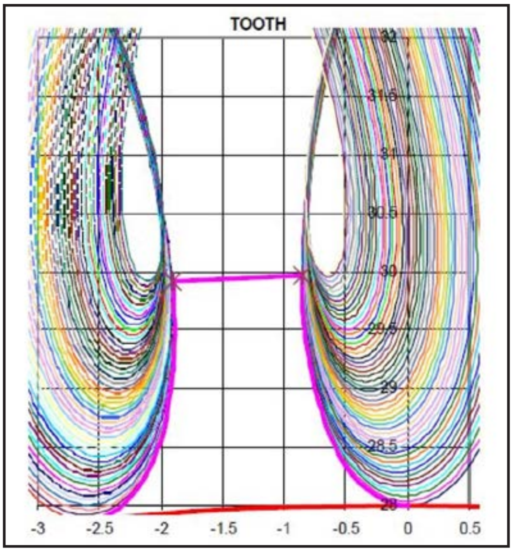

Figure 19 Axial cross-section of the gear tooth: at $z = 0$ mm; lead = 11 mm.

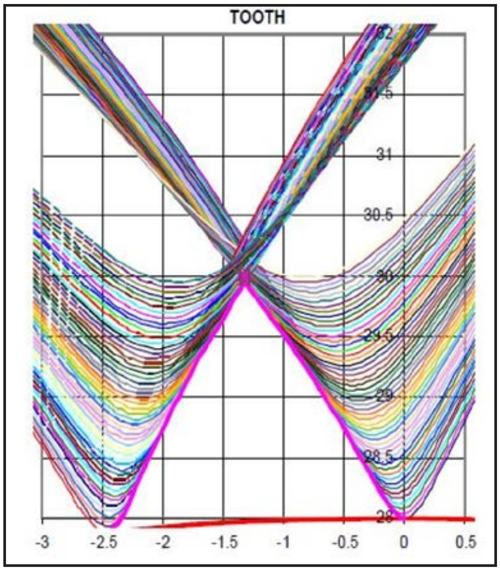

Figure 20 **Axial cross-section of the gear tooth: at $z = 0$ mm; lead = 9 mm.**

and some undercut — especially on the tooth left side. For $z=0$ mm and lead = 9 mm, a cross-section of the tooth is shown (Fig. 20) indicating tooth clipping. With this software the user can design the gear tooth to any desired shape by simply manipulating the values of the design parameters and observing how the shape changes. The pinion of the worm gear will be designed in the same manner as the pinion of the Spiroid gear discussed above.

Spiroid Worm Hybrid Gears

For aerospace, robotic, and medical applications there is an increasing demand for smaller and lighter gear sets with higher torque capability. In gear set design this translates to a need for increased tooth contact and contact ratio. As observed previously, Spiroid and worm gears are suitable for these applications. The same pinion that drives a Spiroid gear can have more teeth driving another Spiroid gear on the same shaft, on the opposite side of the centerline. This immediately doubles the torque capability, yet with a minimal increase in weight and size. The Spiroid gear must be of the skew angle-type so that the added teeth will not interfere with the first gear teeth on the other side of the centerline. This combination is called a "double-Spiroid gear." We also determined that there is an unoccupied space between the two Spiroid gears in their axial direction, and between their shaft and the pinion, where a worm gear can fit nicely. This worm gear can be driven with additional teeth on the pinion shaft between the two sets of teeth driving the double Spiroid gears. Adding the worm gear will add more torque capability, but with a negligible weight increase and no size increase. The pinion will have a tilt angle with the worm gear that is the same as the skew angle it makes with Spiroid gears. This is why the tilt angle was added as a new design parameter in the abovementioned worm gear design (Fig. 21).

Both hob and pinion for the Spiroid and worm gear were designed independently. Excepting the outside radius, the number of teeth (starts) and tooth depth were retained. In this special case, with an RPM ratio of 19, to make a good Spiroid gear its hob lead had to be 13.5 mm; and to get a good worm gear, the lead had to be 10 mm. However, in other cases with exactly the same size but an RPM ratio of 38, it was possible to have an exact geometry of hob and pinion for the Spiroid and the worm gear. This makes gear and pinion manufacturing — and assembly — much easier.

This set of gear and pinion was manufactured by a milling CNC machine; it was possible because the *xyz* data points of the tooth profile and the tool path were available. The design optimization was done in a few hours and machining was completed the next day. A hob was not needed, thus saving its cost — and a great deal of time.

For the Spiroid and worm gear, our software is a tool that, in a matter of a few hours, designs the hob and the gear simultaneously. It also provides accurate calculation of the gear tooth profile in terms of the design parameters in sketches and *xyz* data points. The points can be used to produce a true 3-D drawing of the gear. Knowing the gear and pinion tooth profiles in 3-D sketches and data points enables gear CNC milling or shaping, besides hob manufacturing. Contact points between pinion and gear teeth become known. Clearance between the two teeth surfaces when moving away from the contact points will be known. Calculation of teeth contact spread under load for given materials become possible. Accurate FE and load analysis will also be possible (Fig. 22) that was performed on the hybrid gear. Quality control can also be accurately and easily conducted directly on the gear and the pinion.

Mathematical modeling is a new and revolutionary process for improving the design and manufacturing of all gear

Figure 21 **New "hybrid" gear made of a double-Spiroid gear, and worm gear driven by same pinion.**

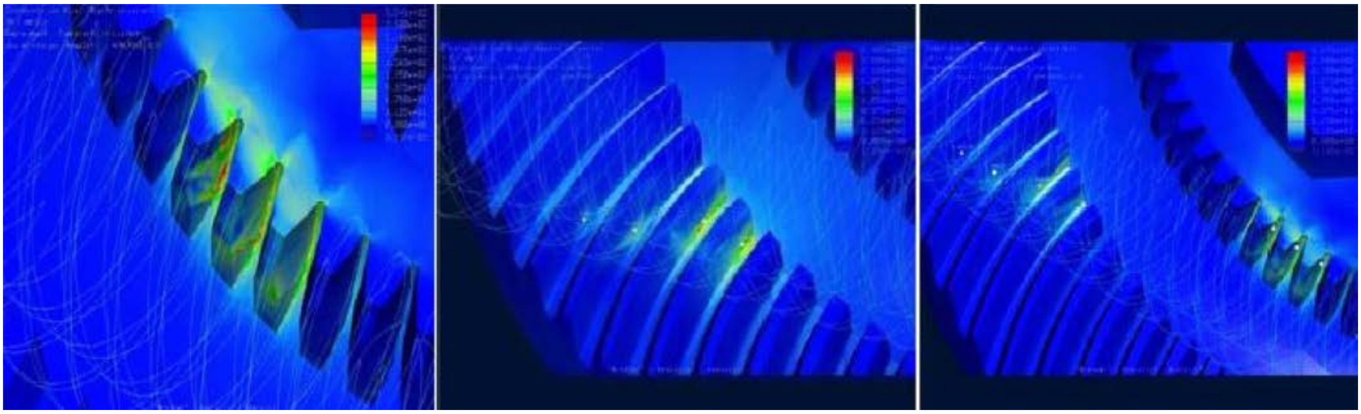

Figure 22 FEA outputs on the hybrid gear.

types — not just the Spiroid and worm gears. It is based on mathematically generating accurate gear tooth profile in terms of the geometry of the tool and the machining process that will be used as input parameters in software. It provides the means for optimization of the parameters in order to obtain the desired gear. The results will be designing the tool and determining the machining set-up to achieve the objectives. The generated gear tooth profile in sketches and *xyz* points can be used to generate the 3-D model of the gear. This opens the door for machining by CNC milling or shaping. With the right approach and diligence it is possible for Spiroid and worm gears to be included in the AGMA standards. There are many improvements and innovations needed in the gear industry, including the spur gear, where mathematical modeling can be the answer.

Spiral Bevel Gears

In this paper we will consider only the spiral bevel tapered hobbing manufacturing method. The gear/pinion assembly or gear/hob set-up is shown in Figure 23; the figure shows the case with no inclination angle.

With an inclination angle, the hob or pinion axis will have a tilt from the horizontal position. The hob geometry is similar to that of a Spiroid tapered hob (Fig. 24) that represents an axial cross-section with straight tooth sides. Therefore the design parameters of the spiral bevel gear are 14 (Table 3).

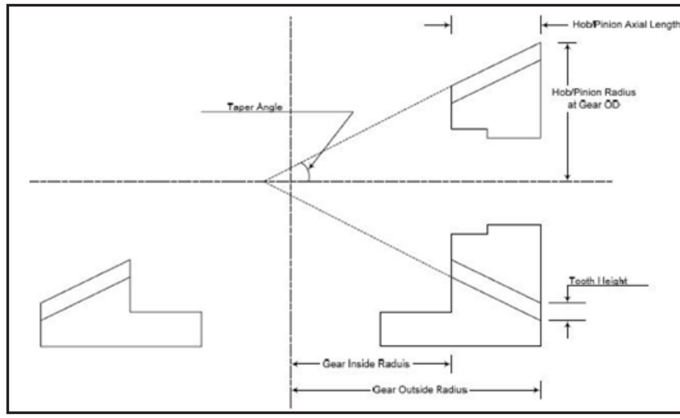

Figure 23 Spiral bevel gear set-up with hob or pinion.

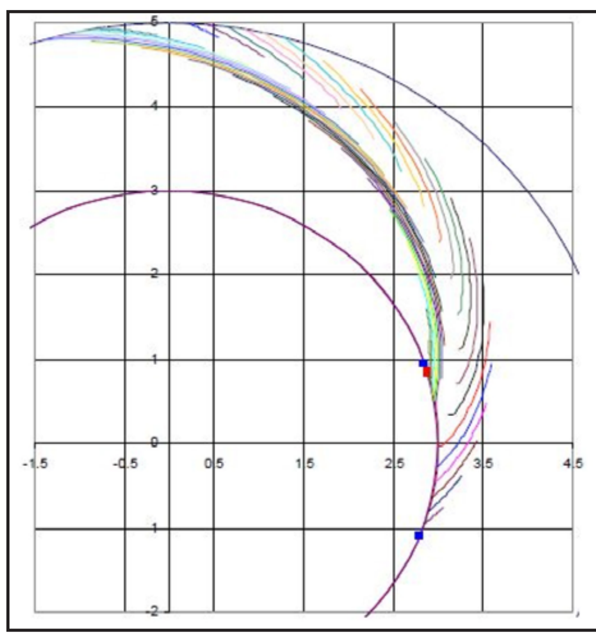

Figure 24 Axial cross-section of spiral bevel gear hob showing its geometry and parameters.

Table 3 Design parameters of spiral bevel gears	
Hob/pinion number of teeth (starts)	9
Hob/pinion radius at gear OD	5.0"
Hob/pinion axial length	2.0"
Tooth height	0.5"
Hob/pinion lead	8.0"
Hob high side pressure angle	30.0°
Hob low side pressure angle	20.0°
Hob tooth width on top	0.18"
Hob/pinion taper angle	30.0°
Hob pinion inclination angle	0.0°
Gear inside radius	3.0"
Gear outside radius	5.0"
Gear number of teeth	9

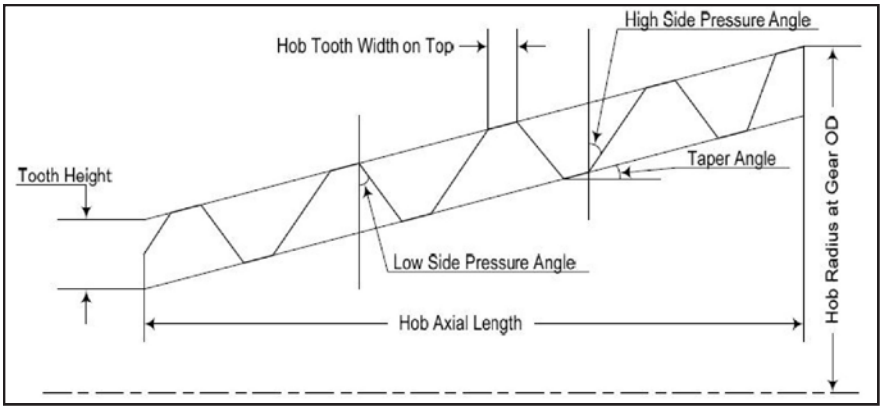

Figure 25 Spiral bevel gear groove with high-side and low-side profiles made of curves in planes perpendicular to the gear axis at different heights.

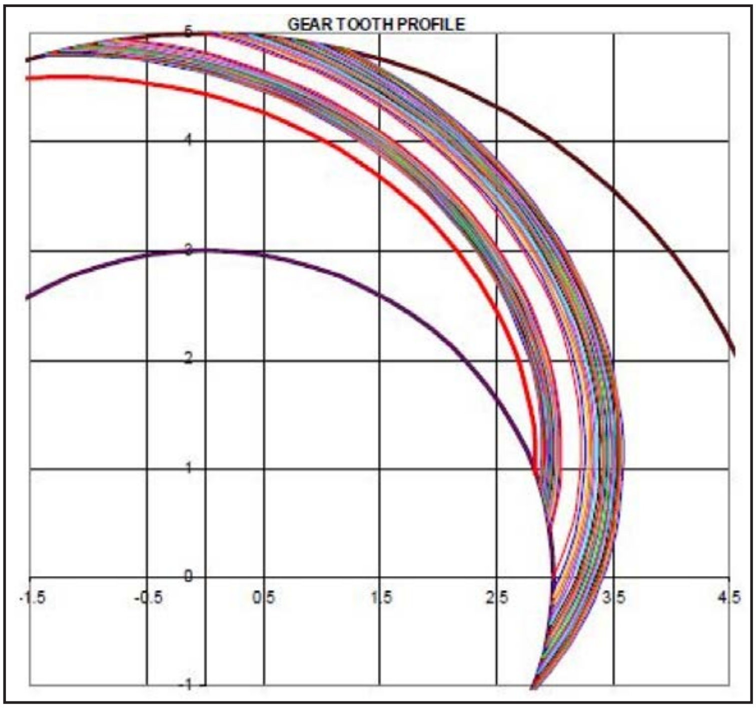

Figure 26 Spiral bevel gear groove with high-side and low-side profiles made of curves in conical surfaces parallel to root or top of the gear.

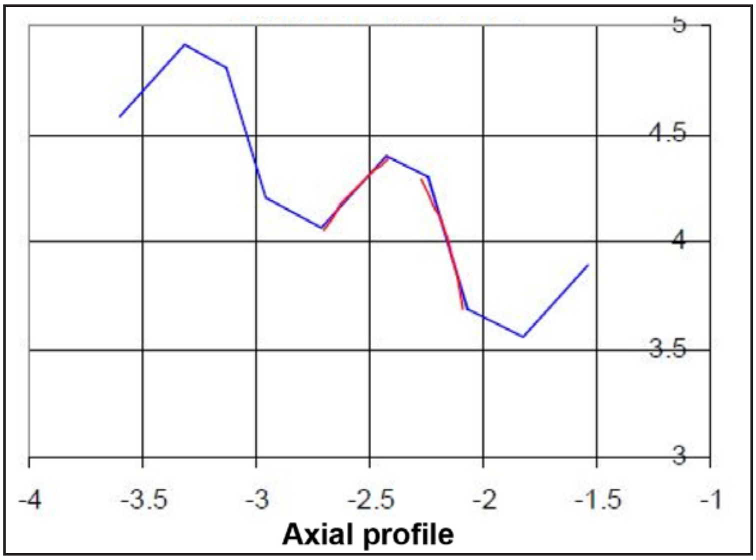

Figure 27 Axial cross-section of the spiral bevel gear hob and pinion.

For illustration we considered designing a set that has an RPM ratio of 1.

The outside and inside gear radii were selected to be 5" and 3", respectively. The outside radius of the hob at gear OD was also selected to be 5". The values of the other parameters were manipulated until a good design was obtained (Table 3).

For gear tooth profile representation we selected a coordinate system where the z axis is aligned with the gear axis. As for the flat-face gear, the spiral bevel gear groove will be sketched (Fig. 25).

The curves on the right represent the profile of the high side, and the curves on the left represent the profile of the low side. Each side is made of 21 horizontal curves in planes perpendicular to the gear axis.

Another representation of the same groove is shown (Fig. 26). Here, too, the 21 curves on the right form the high side and the 21 curves on the left form the low side. But they are arranged in pairs, where the pair in the center is at the root. The other pairs are located at equal increments of the z height above the root; the heavy red line at extreme left is the top of the next groove.

As each curve is made of 21 points, each side has 441 points that can be used to generate the 3-D model of the gear groove, using engineering CAD software. The 3-D model of the gear can be generated by circumferentially copying the groove nine times.

Figure 27 shows a sketch (in blue) of the axial cross-section of the hob. The sketch (in red) is an axial cross-section of the pinion tooth to mate with the gear. It is contained inside the hob tooth and, in this case, it is designed with circular sides that touch the hob sides exactly at tooth mid-height. As a result, under no-load, the manufactured pinion tooth will touch the gear tooth in one point at tooth mid-height.

The design method of the spiral bevel gear presented in this paper may be unique to *Gearometry,* based on using tapered hobbing for manufacturing; there are other established design and manufacturing methods in the industry. The details of design and manufacturing set-up are kept as proprietary company secrets by the manufacturers of the machines and the tools that are used to cut the gears.

Conclusion

Mathematical modeling is a new and unique tool for gear design that allows designers to achieve optimal gear sets. It reduces the time and cost of designing the hob and manufacturing the gear.

But more importantly, for these gears, it numerically and accurately generates the gear tooth profile.

This calculation now makes it possible to produce a true 3-D drawing of the gear; accurately determine the contact point and clearance between the gear/pinion teeth surfaces; conduct FE analysis of the gear set under load; and many other useful analyses. It opens the door for CNC machining, in addition to hobbing, teaching, and the research and development of standards for these gear types. Mathematical modeling can improve the already-valuable Spiroid gear form and make it acceptable in large-size gear markets such as wind energy and large machinery at affordable cost. It also opens the door for many innovations in gear technology. The new Spiroid/worm hybrid gear is one example of such innovations. (*Author's Note*: *See U.S. Patent Application Publication No. 2012/0000305, "Hybrid Enveloping Spiroid and Worm Gear," published January 5, 2012, as to which the author is a co-inventor.*) With mathematical modeling, gear technology innovation is limited only by the imagination of the gear designer and the gear manufacturing machine maker. **PTE**


Dr. Ghaffar Kazkaz is currently a consultant in gear technology specializing in gear geometry and design. He started his consulting company, Gearometry, after leaving ITW Technology in 2011. He has a bachelor in pure math and physics from the Syrian University of Damascus, a doctorate in physics from the University of Grenoble, France and a PhD in electrical engineering from the University of Illinois in Chicago. Kazkaz started gear mathematical modeling at ITW to help ITW's Spiroid division in Spiroid and worm gear design. After retirement he made significant improvements and extended mathematical modeling to spiral bevel and spur gears. (Mathematical modeling is based on calculating the gear tooth profile in terms of the cutting or grinding tool geometry and machining set-up parameters. It also calculates the gear tooth profile in terms of the mating gear tooth profile and assembly conditions.) He has developed software formatted in Excel sheet programs for gear design and is looking to work with a team to market the software. Kazkaz is also interested in designing gears and developing custom software in gear design and manufacturing for individual companies.

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