Gears for Hydraulic Pumps: Development and Results

A procedure to reduce dimensions and mass of gear pumps

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After a brief introduction to the importance of gear pumps in internal combustion engines, as well as in the most diverse hydraulic applications, a calculation method was applied that allows for sizing with considerably higher delivery rates. Upon identifying and analyzing a traditional pump, along with two construction solutions of asymmetric gear pumps ($\tau \neq 1$), we then compared their related performances.

Introduction

External gear pumps are the most common primary hydrostatic units in the field of hydraulics and in the automotive sector (internal combustion engines and other services); approximately 90 million units are built annually. For that reason they are the object of continuous studies on the part of numerous researchers and company study centers. Said studies focus on involute profile and on the details (bearings of the wheel axis and shaft, axial covers, etc.) making up the pump itself, with the aim to improve performance and, in particular — increasing the delivery rate. These studies were conducted on external gear pumps featuring a driving wheel and a driven wheel with the same characteristics and, therefore, with the same number of teeth. The number of teeth of the two wheels is approximately: 13 – 13 teeth; 12 – 12 teeth; or 11 – 11 teeth. As known, on equal factors the pump's delivery rate increases as the number of teeth decreases; for that reason we find pumps also with 10 teeth per wheel or even 9 teeth per wheel. In the latter case, however, the wheels have a large undercut, or heart-shaped teeth. One aspect that should be explored further is to examine what happens if we study a gear pump composed of two gear wheels — each with a varying number of teeth — i.e., with a gear ratio other than one (asymmetric pumps).

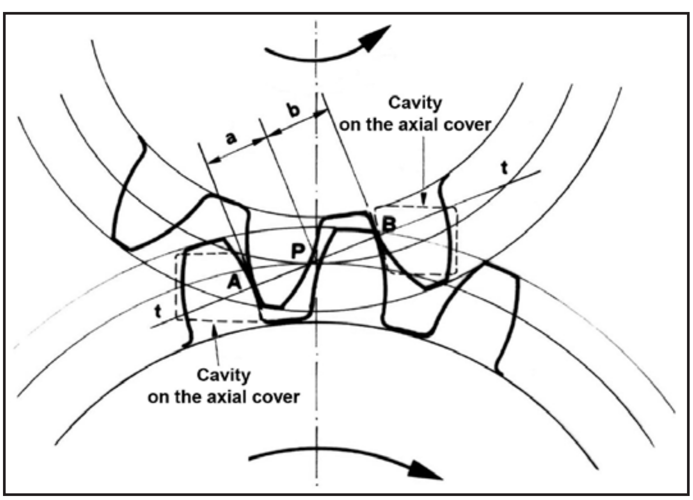

Figure 1 **A study of function D (average delivery rate) based on the characteristics of the gear wheels, will produce a functional relation that shows the parameters determining the delivery rate.**

Gear Wheels with Gear Ratio ≠ 1

In order to study the impact of gear ratio on pump delivery rates, we must establish a relationship between delivery rate and gear wheel characteristics. If we study function D (average delivery rate) based on the characteristics of the gear wheels, we obtain a functional relation that shows the parameters determining the delivery rate.

This study relies on two basic assumptions (Fig. 1): a) the length of the arc in which a pair of teeth guarantees that the sealing of oil under pressure is equal to the pitch ($a+b=p$); and b) the cavities on the axial covers are positioned in such a way that the distance from the tangent point between the pitch circles of the contact point between the teeth of the pair of teeth coming out of the sealing arc is equal to the contact point between the teeth of the pair is entering the sealing arc ($a=b$). Based on the foregoing, we obtain (Ref. 2) a complex relationship expressing the average delivery rate D, based on the parameters of the gear wheels; said relationship is, for brevity reasons, set out as a function:

$$D = b\,\omega_1\,a'^2\,\{\psi_1, \psi_2, \psi_3, \psi_4\} \tag{1}$$

where:

$$\psi_1 = \frac{\gamma}{2}\left(\frac{\cos^2\alpha'}{\cos^2\alpha_0} - 1\right)$$

$$\psi_2 = \frac{2\gamma^2\cos^2\alpha'}{z_1^2\cos^2\alpha_0}\left[(1+x_1-k_1)^2 + \frac{\gamma}{1-\gamma}(1+x_2-k_2)^2\right]$$

$$\psi_3 = \frac{2\gamma^2\cos^2\alpha'}{z^1\cos^2\alpha_0}(2+x_1+x_2-k_1-k_2)$$

$$\psi_4 = \frac{\pi^2\gamma^2\cos^2\alpha'}{6(1-\gamma)z_1^2}$$

$$\gamma = \frac{z_1}{z_1+z_2}$$

With (symbols are in accordance with international standardization):

D = average delivery

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a' = center distance
α_0 = reference pressure angle
α' = operating pressure angle
z_1, z_2 = number of teeth of gear wheel 1 and of gear wheel 2
ω_1 = angular speed of driving wheel
b = axial face width
x_1, x_2 = addendum modification coefficient of wheel 1 and wheel 2
k_1, k_2 = addendum reduction of wheel 1 and wheel 2

The referenced relationship (Ref. 1) indicates that there are numerous parameters determining the average delivery rate and an even greater number of combinations that can be obtained by varying each of the various parameters. Note, however, that only a finite number of values can be obtained for the delivery rate *D*, as these correspond strictly to practicable solutions, taking into account the construction restraints (undercut, minimum circular tip thickness, contact ratio, etc.). The relationship thus obtained makes it possible to identify the delivery rate *D* both in the case where the number of teeth of the wheels vary and also in the particular case where the wheels have the same number of teeth ($z_1 = z_2$), an equal addendum modification ($x_1 = x_2$), and an equal addendum reduction ($k_1 = k_2$); in other words, the mathematical expression (Ref. 1) makes it possible to calculate the value of *D* also in the particular case of pumps currently in use (identical gear wheel pumps). The relationship thus obtained is therefore of an entirely general nature and allows to calculate the delivery rate *D* for the widest range of possible cases. (*Note*: *Volumetric and hydro-mechanical efficiency are not considered, because they do not depend on asymmetry*.)

Symmetric and Asymmetric Pumps

The foregoing is relevant, when conducting a study, in making comparisons between gear pumps currently in use (symmetric pumps) and pumps with gear wheels varying between them (asymmetric pumps). First, however, a number of parameters need to be set, which, for the sake of fair comparison, must be the same for symmetric pumps as for asymmetric pumps: same center distance, same axial face width, same number of revs of the driving wheel. The first and the second parameter imply that the comparison between symmetric pumps and asymmetric pumps is made on equal overall dimensions.

Study of an Asymmetric Pump

First, we identified a symmetric pump available on the market. The main features of the gear are set out in Table 1, where:

j_p = backlash at the pitch diameter
j_r = top clearance
s_a = circular tip thickness
ε = contact ratio
C = capacity/rev (proportional to the average delivery rate)

Based on the data contained in Table 1, we have then determined the size of the asymmetric pump having the same number of revolutions as the driving wheel; the same axial face width; the same center distance (practically the same radial dimensions); the same pressure angle α_0; and the same backlash j_p at the pitch diameter. Implementing an optimiza-

Table 1 The main features of a symmetric pump gear

	Symmetric Pump	
a'	65.31 mm	
b	10.00 mm	
α_0	20°	
J_p	0.25 mm	
	Driving wheel 1	**Driven wheel 2**
z	9	9
m	6.5 mm	6.5 mm
r_a	39.64 mm	39.64 mm
x	+0.67	+0.67
k	+0.07	+0.07
J_r	0.176 mm	0.176 mm
s_a	0.72 mm	0.72 mm
ε	1.14	
$C = 29.82\ cm^3/rev$		

Table 2 Analysis of study results

	Asymmetric Pump 1	
a'	65.31 mm	
b	10.00 mm	
α_0	20°	
J_p	0.25 mm	
	Driving wheel 1	**Driven wheel 2**
z	11	8
m	6.5 mm	6.5 mm
r_a	44.58 mm	35.16 mm
x	+0.18	+0.36
k	-0.18	-0.05
J_r	0.150 mm	1.339 mm
s_a	1.32 mm	0.81 mm
ε	1.28	
$C = 36.82\ cm^3/rev$		

Table 3 Analysis of study results

	Asymmetric Pump 2	
a'	65.31 mm	
b	10.00 mm	
α_0	20°	
J_p	0.25 mm	
	Driving wheel 1	**Driven wheel 2**
z	10	9
m	6.5 mm	6.5 mm
r_a	41.71 mm	38.47 mm
x	+0.24	+0.30
k	-0.18	-0.12
J_r	0.150 mm	0.899 mm
s_a	0.72 mm	0.72 mm
ε	1.32	
$C = 34.52\ cm^3/rev$		

tion procedure that we have previously fine-tuned (Ref. 2), we have identified each of the numerous solutions that can actually be implemented. Among these, the most convenient from a performance and technological viewpoint can be selected. For this particular case we have provided two possible project solutions (Tables 2 and 3).

Analysis of Study Results

Both of the solutions set out in Tables 2 and 3 are practical from a constructive viewpoint, and both are more convenient in terms of performance than a symmetric pump of equal dimensions. The most convenient solution from a technological and operational viewpoint is solution No.1 (Table 2). In fact, this solution allows for a considerable increase in delivery rate compared to the related symmetric pump. The delivery rate of the asymmetric pump No. 1 is increased by 23.47%, compared to that of the symmetric pump.

Size Reduction on Equal Delivery

Moreover, based on these results, it can be stated that on equal delivery rate of the symmetric pump, it is possible to construct an asymmetric pump with the same delivery rate as the symmetric one — but with smaller radial dimensions or with smaller axial dimensions or with smaller radial *and* axial dimensions. It is possible, in other words, to construct an asymmetric pump with the same delivery rate but with a much smaller axial face width than that of the symmetric pump (pump with smaller axial dimensions) on equal radial dimensions. As an alternative, it is possible to construct an asymmetric pump with the same delivery rate but with much smaller radial dimensions than those of the symmetric pump on equal axial face width. Or, finally, it is possible to construct an asymmetric pump distributing the size reduction in part on the radial dimensions and in part on the axial dimensions. The foregoing considerations may be of particular relevance in the field of hydraulics and, to an even greater extent, in the field of internal combustion engines used in motor vehicles, where problems of weight and dimensions have great impact.

Conclusions

Based on the analysis set for the study of the increase of delivery rate of a gear pump, we may conclude that significant results can be obtained by using gear ratios other than one. In fact, on equal dimensions of a traditional gear pump (gear ratio equal to one) it is possible to dramatically increase the delivery rate by adopting a gear ratio determined by the procedure illustrated in the study. Conversely, it is possible to construct an asymmetric pump with the same delivery rate, but with reduced (radial and/or axial) dimensions, and with a corresponding reduction of mass.

From a technological viewpoint, we are dealing with holes in the pump body (which are to house the two different gears) with a varying, rather than equal, diameter. This was seen in the case of symmetric gears, as the machining allowance of the two different holes on the pump body was brought to size on the two holes through the use of tools having a distance from the center-of-rotation equal to that of symmetric gears and, hence, without need to modify either the machinery or the manufacturing process. Another considerable advantage is the possibility, with a single asymmetric pump, of having two deliveries, thus making the driving wheel the larger wheel or the smaller wheel. In such a circumstance two deliveries are obtained (only one of which is optimized) in which the values have the same ratio as those between the number of teeth of the two wheels. As a result, it is possible to have an entire set of delivery rates through a number of asymmetric pumps equal to half the symmetric pumps. This implies lower production costs due to a simpler and smaller structural make-up of the pump — when compared to the traditional assembly — as well as lower warehousing and distribution costs. **PTE**

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